

ABSTRACT

Title of dissertation: ESSAYS ON BANK CAPITALIZATION
AND MACROECONOMIC FLUCTUATION

Xing Hong
Doctor of Philosophy, 2019

Dissertation directed by: Professor John Shea
Department of Economics

This thesis provides two studies in the relationship between bank capitalization and macroeconomic fluctuations. In the first chapter, I study the effect of bank capital shortfalls on macroeconomic fluctuations through changes in lending standards. Existing literature has primarily focused on the rise of credit spreads when banks suffer capital losses. In addition to this standard interest rate channel, this paper innovates by introducing a new credit rejection channel - denying more loan applications (tightening lending standards) - into a macro model with financial frictions. The model features an endogenous time-varying risk threshold for credit rejection, which in turn is linked to banks' balance sheet conditions. I incorporate the rejection mechanism into a quantitative general equilibrium model and conduct a banking crisis experiment. During the crisis, loan rejection rates rise significantly, and lending rate spreads increase mildly, which are consistent with observations on the bank loan market during the Great Recession. The simulation results further show that the model with this new channel generates larger amplification of macroeconomic variables, compared to an otherwise identical benchmark model. This result

is driven by a combination of two forces: a decline in loan volume and a shift in the composition of banks' lending pool, as banks reallocate funds away from risky firms. Given that riskier firms tend to have better growth prospects, such reallocation can have long-lasting scarring effects on the economic recovery.

In the second chapter, we take a normative angle of bank capital analysis. We develop a quantitative dynamic stochastic general equilibrium model to identify bank capital gaps (deviations of the observed level from the optimum) and to shed light on regulatory policies regarding capital requirement. We propose a tractable model that includes firms' and banks' choice on joint capital structure, and their endogenous default caused by idiosyncratic and aggregate risk. The model is estimated using Bayesian methods with quarterly data on US macroeconomic and financial variables spanning from 1991Q1 to 2016Q4. Our counterfactual analysis shows that the impulse responses in the optimal economy exhibit smaller magnitude compared to that in the calibrated economy. We further decompose the historical fluctuations in bank capital gaps into contributions from a series of financial shocks, in addition to the standard macroeconomic shocks. We find that the aggregate risk shock plays an important role in explaining the spike in capital gaps during the 2007-09 financial crisis. Capital gaps lead to (i) excessive increases in banks' default risk and cost of funding, (ii) gaps in lending, investment, employment and output.

ESSAYS ON BANK CAPITALIZATION
AND MACROECONOMIC FLUCTUATION

by

Xing Hong

Dissertation submitted to the Faculty of the Graduate School of the
University of Maryland, College Park in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
2019

Advisory Committee:
Professor John Shea, Chair
Professor Borağan Aruoba
Professor Felipe Saffie
Professor Luminita Stevens
Professor Phillip Swagel

© Copyright by
Xing Hong
2019

Dedication

To Lichen, and my parents.

Acknowledgments

I deeply thank all the people who gave me support, advice and encouragement and made this thesis possible.

I am especially indebted to my main adviser, John Shea. He enlightened me in a variety of academic dimensions, from exploring innovative research ideas to developing and polishing theories. Without his continued patience and constructive guidance, I would not have overcome many challenges in the development of this thesis. I thank wholeheartedly, for his detailed suggestions throughout this process.

I am also incredibly grateful to my other advisors. Borağan Aruoba has always made him available for me when I seek for advice. He illuminated me not only through his excellence in technicality, but also in the way of thinking critically as an economist. Felipe Saffie never hesitated to share his insightful advice. He has given me an incredible amount of concrete suggestions, which have laid the foundation for the completion of this thesis. I also thank Luminita Stevens for much helpful advice and comments. She taught me how to conduct research when I embarked on my research journey during my third year, and also how to be an effective instructor throughout my year-long TA experience with her.

Besides my advisors, I would also want to express my gratitude to Guido Kuersteiner. His advice was tremendously helpful in navigating through the job market. I thank Phillip Swagel for willingly serving on my committee.

Profound gratitude go to Romain Ranciere and Fabian Lipinsky, my two supervisors at the International Monetary Fund. I am grateful for the opportunity of

working with them, which vastly improved my understanding in policy designs. I also enjoyed the effective intellectual discussion with my colleague and friend Malgorzata Skibinska at every stage of our research project.

My journey as a doctorate student would not be this exciting without the companionship of my PhD cohorts at the University of Maryland. I thank all my friends, Jake Blackwood, Hyung Choi, Joonkyu Choi, Diyu Guo, Bryan Hardy, Rodrigo Heresi, Lin Hong, Shen Hui, Karam Jo, Ernest Koh, Edith Laget, Tzu-Yao Lin, Hidehiko Matsumoto, Sungho Noh, Svetlana Pivovarova, Cristian Sanchez, Xuezhen Tao, Jikun Wang, Hsuan Yu and Youngjin Yun among others.

Last but foremost, I owe my deepest thanks to my family for their unconditional love. I would like to thank my girlfriend, Lichen, for her love and trust. Nothing would be possible without her support. My mother and father would always do anything they can to support me. I thank them with all my heart for everything they gave me.

Table of Contents

Dedication	ii
Acknowledgements	iii
Table of Contents	v
List of Tables	vii
List of Figures	viii
1 Bank Capitalization, Lending Standards, and Macroeconomic Dynamics	1
1.1 Introduction	1
1.2 Related Literature	6
1.3 Lending Standards in the Partial Equilibrium Model	10
1.3.1 Firm Heterogeneity in Risk	11
1.3.2 Thresholds of Firm Default	12
1.3.3 Bank Revenue	14
1.4 The General Equilibrium Model	22
1.4.1 Entrepreneurs	23
1.4.1.1 Technology	23
1.4.1.2 Labor Choice and Firm Value	24
1.4.2 Banks	26
1.4.2.1 Loan Portfolio and Bank Default	27
1.4.3 The Mutual Fund	29
1.4.4 The Financial Contracts	30
1.4.5 Households	32
1.4.6 New Capital Producers	33
1.4.7 Equity Dynamics	34
1.4.8 Market Clearing	36
1.5 Calibration	36
1.5.1 Choosing Parameters	36
1.5.2 Parameter Identification	41
1.6 Numerical Results	42
1.6.1 Dynamic Effects of Financial Shocks	44
1.6.2 Comparison with the Benchmark Model	46
1.7 Conclusion	51

2	Capital Gaps, Risk Premia Dynamics and the Macroeconomy (coauthored with Fabian Lipinsky and Malgorzata Skibinska)	54
2.1	Introduction	54
2.2	Literature Review	58
2.3	The Model	59
2.3.1	Households	59
2.3.2	Firms	60
2.3.2.1	Firm Production	61
2.3.2.2	Firm Value	62
2.3.2.3	Firms default	63
2.3.3	Financial Intermediaries	65
2.3.4	The Mutual Fund	66
2.3.5	Joint Capital Structure Choice	67
2.3.5.1	Case I: Optimal Capital Structure	67
2.3.5.2	Case II: Law of Motion for Equity	69
2.3.6	New Capital Producers	72
2.3.7	Market Clearing	73
2.4	Parameterization	73
2.4.1	Calibrated Parameters	76
2.4.2	Estimated Parameters	76
2.5	Quantitative Results	78
2.5.1	Impulse Responses	79
2.5.2	Historical Decomposition	84
2.6	Conclusion	85
A	Appendix for Chapter 1	89
A.1	Figures of Motivating Facts	89
A.2	Financial Contracts in the Full Model	93
A.3	Optimality Conditions	95
A.4	Variable Definition	100
A	Appendix for Chapter 2	102
A.1	Equilibrium Conditions for Optimal Economy	102
A.2	Equilibrium Conditions in the Calibrated Economy	103
A.2.1	Equilibrium Conditions	104
	Bibliography	108

List of Tables

1.1	Calibration I: Standard Parameters	37
1.2	Calibration II: Internally Calibrated Parameters and Moments	38
2.1	Calibrated Parameters	76
2.2	Priors and Posterior Estimates for the Model Parameters	78

List of Figures

1.1	The Tradeoff of Increasing Lending Rate on Bank's Revenue	16
1.2	Bank's Expected Revenue as a Function of Lending Rate	18
1.3	Banks' Expected Revenue for Different Borrowers' Risk Levels	21
1.4	Simulated Method of Moments	43
1.5	IRFs to Financial Shock - Bank Variables	45
1.6	IRFs to Financial Shock - Aggregate Variables	47
1.7	IRFs to Financial Shock - Comparison	50
2.1	Firms' Balance Sheet	64
2.2	Banks' Balance Sheet	66
2.3	Data Series for Estimation	75
2.4	Impulse Response Functions to Idiosyncratic Risk Shock	81
2.5	Impulse Response Functions to Aggregate Risk Shock	82
2.6	Impulse Response Functions to Productivity Shock	83
2.7	Historical Decomposition of De-trended Financial Funding Spreads	85
2.8	Historical Decomposition of De-trended Corporate Credit Spreads	86
2.9	Historical Decomposition of Bank Capital Gap	87
A.1	Measures of credit accessibility for small businesses	90
A.2	Interest rates on Commercial and Industrial loans and Treasury Bill	91
A.3	Bank loan rejection rates for U.K. 2001-2012	92

Chapter 1: Bank Capitalization, Lending Standards, and Macroeconomic Dynamics

1.1 Introduction

The last global financial crisis confirmed the vital role of bank capital buffers for financial stability. Losses in the banking sector caused large contractions in the supply of credit to the corporate sector.¹ In the United States, business lending of all commercial banks declined by about 34% from the last quarter of 2008 to the third quarter of 2010.² Concurrent with the collapse of bank lending, however, interest rate spreads on bank loans remained almost flat during the same period.³ This poses a challenge for existing models of the transmission channels of bank losses on credit supply, which largely focus on price channels, and suggests that other channels may also be important in accounting for the fluctuations in bank lending. Indeed, banks tightened their lending standards significantly, and rejected

¹See, e.g., [Ivashina and Scharfstein \[2010\]](#), [Adrian et al. \[2013\]](#), [Chodorow-Reich \[2014\]](#).

²This fact is calculated using data on all commercial and industry loans, all commercial banks from the Board of Governors of the Federal Reserve System's E.2 release.

³For example, the spread between the effective loan rate on C&I loans and the 3-Month Treasury Bill rate was 2.35% on average during the 18-month crisis period with a peak value of 3%, whereas this spread was on average 2.33% for the 18-month period prior to the crisis with a peak value of 2.89%. This fact is calculated using data from the Board of Governors of the Federal Reserve System's E.2 release.

more loan applicants during the crisis.⁴ [Bentolila et al. \[2018\]](#) document that banks mainly responded to balance sheet deterioration by denying more loan applicants, whereas the interest rate was scarcely used to ration credit demand during that period.⁵ Yet relatively little is known regarding the mechanism through which the health of banks' balance sheets affects lending standards and in particular banks' decisions on credit rejection, the outcome of which has implications not only on the volume but also on the composition of credit.

In this paper, I propose a new transmission channel linking bank capital shortfalls to the supply of credit in a dynamic stochastic general equilibrium model – *the credit rejection channel*. Through the lens of the model, I attempt to address the following three questions. 1). Why is there credit rejection? In other words, what motivates banks to reject some borrowers entirely, rather than charging riskier borrowers a higher interest rate? 2). How are banks' decisions on credit rejection affected by the health of their balance sheets? 3). What are the implications of this mechanism for the allocation of credit, and more importantly, for macroeconomic outcomes?

Credit rejection, or equivalently credit rationing, is a special phenomenon that has attracted much attention in the literature.⁶ Although there have been a number of theories offering rationales for this phenomenon, these theories are either in a stylized or static setting, which makes it difficult to draw quantitative implications, or lack a meaningful capital structure of banks, which makes it difficult to

⁴See Appendix A.

⁵Similar findings are documented in [Rodano et al. \[2018\]](#) for the Italian banking sector.

⁶For example, see [Jaffee and Russell \[1976\]](#), [Stiglitz and Weiss \[1981\]](#), [Bester \[1985\]](#), and [Williamson \[1987\]](#).

address questions related to bank health. This paper tackles both challenges. I first develop a micro-founded theory in which credit rejection is an equilibrium outcome in the loan market. To this end, the model generates an endogenous cut-off lending rule, in which banks set a maximum acceptable level of borrowers' risk and deny applications for borrowers with risk levels higher than this threshold. The key to this result is the modeling of firms' endogenous default as a function of banks' lending rate, which implies a risk/return trade-off for banks in adjusting lending rates and breaks the monotonic relationship between lending rates and banks' revenue. I further incorporate this mechanism into a tractable quantitative general equilibrium framework. Through the modeling of banks' endogenous default, I establish a link between conditions on banks' balance sheets and banks' decisions on credit rejection.

The first contribution of this paper is the modeling of credit rejection. The credit market setting in this paper follows [Bernanke et al. \[1999\]](#) (henceforth, BGG), in which a financial contract is specified between borrowers (entrepreneurs) and lenders (banks). I add additional features to the BGG by allowing entrepreneurs to be heterogeneous in terms of their ex-ante *risk*. In this environment, I analyze how borrowers' risk affects banks' profit from lending. The first result is that a bank's expected revenue is an inverse U-shaped function of the lending rate. This is due to the risk/return tradeoff faced by the bank in charging a higher lending rate: higher lending rates increase the loan repayment from borrowers who do not default; however, they also increase the fraction of borrowers that default and consequently

the associated loss from default.⁷ This concave relationship implies that there exists a cap on banks’ expected revenue for any given loan – the bank can never earn more than that cap. Moreover, I show that this maximum expected revenue monotonically decreases with borrowers’ risk. The intuition is straightforward. Suppose borrowers’ risk increases in a mean-preserving manner. The increase in right-tail risk does not benefit the bank, as the loan repayment is predetermined. Yet the increase in left-tail risk hurts the bank because it increases the borrower’s probability of default and hence lowers the bank’s expected return. For this reason, if the borrower is too risky, the bank’s maximum expected value can fall below the bank’s cost of funds. This is when the bank starts to reject borrowers. The threshold value on the borrower’s risk is interpreted as the bank’s lending standard.

Another novel feature of this paper is the presence of equilibrium default of both firms and banks in a joint framework, where both defaults are caused by fundamental productivity shocks to firms.⁸ In the model, firms’ return is subject to an idiosyncratic productivity shock and an aggregate productivity shock. The default of firms can be attributed to either shock, or a combination of both. In contrast, while banks are able to diversify idiosyncratic shocks through lending to a large number of firms, there is no way for banks to diversify the aggregate shock,

⁷In a similar spirit, this inverse U-shaped relationship is also featured in [Stiglitz and Weiss \[1981\]](#). Their result is due to adverse selection: borrowers have private information on their projects. Therefore, when banks post a higher loan rate, they only attract bad borrowers, which thus lowers the expected return. In contrast, I do not assume ex-ante information asymmetry in this paper. My result is driven by the default cost implied by the debt contract.

⁸Methodologically, the modeling of firm default follows [Christiano et al. \[2014\]](#) and henceforth [Bernanke et al. \[1999\]](#). While other models also feature equilibrium default of firms and banks (e.g., [Clerc et al. \[2015\]](#), [Gete \[2018\]](#)), bank default in these models is triggered by some “profitability” shock to banks. In contrast, bank default in my model is driven by fundamental productivity shocks to firms.

and as a result bad aggregate shocks trigger bank default. I introduce a capital structure choice in banks' balance sheets: they borrow from a mutual fund in the form of debt, and accumulate equity over time. The possibility of default makes banks' debt risky and leads to positive interest rate spreads on banks' debt. It is precisely this feature of bank default that transmits changes in banks' balance sheet conditions to banks' cost of funds, and therefore to banks' lending decisions.

I then calibrate the model to the U.S. economy with data on the U.S. banking sector. I match model objects in the steady state with key credit market indicators in the data, including the loan rejection rate, firms' and banks' probability of default (PD), the loss given default (LGD) of bank loans, and non-financial firms' loan spreads. The model overall provides a good fit to the data. I simulate a crisis episode triggered by exogenous negative shocks to banks' capital. A shortfall in banks' capital increases banks' leverage and the likelihood of bank default, which leads to a higher cost of funds for banks. Since banks' revenue decreases with the average risk in the pool of borrowers, banks choose to lower the risk threshold to increase their expected revenue. Consequently, the fraction of rejected borrowers rises and total credit supply declines. Aggregate investment, employment, and output subsequently fall.

The model with this new rejection channel (the full model) is compared to an otherwise identical benchmark model. Interestingly, while the interest rate channel is still present in the full model, it reveals dampened effects. The impulse responses of firm credit spreads in the full model show only mild increases following negative financial shocks, in comparison to the benchmark model. This response of interest

rates in the full model is driven by a combination of two forces. On the one hand, since the rejected borrowers are the riskier ones, an increase in credit rejection lowers the average risk in the pool of successful applicants, which puts downward pressure on the credit spread. On the other hand, the pass-through of rising bank funding costs puts upward pressure on the credit spread. As a result, the interest rate spread on loans increases, but to a lesser extent than it does in the benchmark model. This result is consistent with the fact that interest rate spreads were relatively flat during the recent crisis. More importantly, the simulation results also show that the full model generates larger amplification and propagation of financial shocks compared to the benchmark model.

1.2 Related Literature

This paper is related to several strands of literature. Methodologically this paper contributes to the long line of theoretical and quantitative work on the macroeconomic effects of financial frictions. Earlier work in this literature has posited constraints on the balance sheets of non-financial borrowers. Financial frictions manifest in the form of collateral constraint as in [Kiyotaki and Moore \[1997\]](#), or in a costly state verification (CSV) problem as in [Bernanke et al. \[1999\]](#), [Carlstrom and Fuerst \[1997\]](#), and [Christiano et al. \[2014\]](#). In both setups, adverse shocks to non-financial firms' net worth can be amplified by worsening financial conditions (the financial accelerator). I follow the CSV approach in this paper. This modeling approach generates default in equilibrium and hence credit spreads, which are

absent in collateral constraint models. Since the onset of the Great Recession, a growing body of research has started to focus on frictions arising from the financial intermediation sector. Constraints on the balance sheet of financial intermediaries inhibit the efficient intermediation of funds to the non-financial sector ([Gertler and Kiyotaki \[2010\]](#), [Gertler and Karadi \[2011\]](#), [Gertler and Kiyotaki \[2015\]](#), [Nuño and Thomas \[2017\]](#))⁹. This paper connects these two bodies of research. I add a second layer on top of [Bernanke et al. \[1999\]](#). In the model, the CSV friction exists not only between firms and banks as in [Bernanke et al. \[1999\]](#), but also between banks and banks' debtors. This provides a comprehensive framework to analyze the joint dynamics of borrowers' (firms) and lenders' (banks) balance sheets. One strength of this model is its ability to identify separately banks' and firms' default frequencies and risk premia within a unified framework.

This paper is also related to the literature explaining banks' countercyclical lending standards and exploring its macroeconomic implications. [Ruckes \[2004\]](#) attributes variation of lending policies to changes in the credit quality of borrowers in a model with price competition among lenders. [Dell'ariccia and Marquez \[2006\]](#) argue that changes in the informational structure of loan markets can lead to fluctuations in lending standards. Later work focuses on business cycle effects of lending standards in quantitative general equilibrium models (e.g., [Figueroa and Leukhina \[2015\]](#), [Ravn \[2016\]](#), [Hu \[2017\]](#), [Gete \[2018\]](#)). My main contributions to this litera-

⁹Another part of this literature stress the nonlinear effects of equity losses for intermediaries using continuous time models (See, e.g., [He and Krishnamurthy \[2013\]](#) and [Brunnermeier and Sannikov \[2014\]](#)). In [Brunnermeier and Sannikov \[2014\]](#), adverse shocks to intermediary equity are amplified through fire sales. In [He and Krishnamurthy \[2013\]](#), amplification operates through a substitution of equity financing toward debt financing.

ture are as follows. First, while these papers study how changes in macroeconomic conditions affect bank lending standards, this paper focuses on bank balance sheet conditions, which were at the core of the Great Recession. The results of my model suggest that bank losses can trigger tighter lending standards, even if macroeconomic conditions remain unchanged. Second, this paper connects bank lending standards with borrowers' heterogeneity in risk. This is new to the literature, as previous papers emphasize firms' heterogeneity in productivity and size. An exception is [Geter \[2018\]](#), which studies labor misallocation caused by firms that are denied credit. By contrast, this paper focuses on a different channel through capital, as in the financial accelerator mechanism, and we assess the model's ability to explain observed fluctuations in investment and other macroeconomic variables.

This paper emphasizes the important role of bank capital in transmitting adverse shocks arising from the financial sector to the rest of the economy. Along the same line, [Iacoviello \[2015\]](#) estimates that financial shocks originating from banks accounted for about two-thirds of the decline in output during the Great Recession. Using a monetary model including a BGG financial accelerator mechanism, [Christiano et al. \[2014\]](#) find that agency problems associated with financial intermediation have accounted for a substantial portion of business cycle fluctuations in the US since the 1980s. However, much of the literature has focused on transmission through price effects (e.g., [Ajello and Tanaka \[2017\]](#), [Bigio \[2015\]](#), [Gertler and Kiyotaki \[2015\]](#), [He and Krishnamurthy \[2013\]](#), [Brunnermeier and Sannikov \[2014\]](#)). This paper complements this literature by introducing a non-price mechanism, and I find that the credit rejection channel is quantitatively important.

The mechanism of this paper is supported by a number of empirical papers. Credit rejection is not an unusual phenomenon in credit markets. During the recent financial crisis, for example, [Laufer and Paciorek \[2016\]](#) document that US mortgage lenders introduced progressively higher minimum thresholds on FICO credit scores for approving mortgage loans. [Montoriol-Garriga and Wang \[2011\]](#) use loan-level data in the US and find that the decline in loan supply during the Great Recession was concentrated on loans to the riskiest borrowers. Their findings further reveal that interest rate spreads on small loans declined on average relative to spreads to large loans, which is consistent with the pattern of differentially more rationing of credit to small borrowers. [Bentolila et al. \[2018\]](#) identify a significant causal effect of declines in bank health on employment losses in Spain. They find that interest rates were scarcely used by weak banks to ration credit. Rather, the acceptance rate of loan applications declined significantly following the bank crisis, and the pattern was more pronounced for weak banks than for healthy banks.¹⁰ A similar result is found in Italian credit markets. [Rodano et al. \[2018\]](#) find that most Italian banks adjusted their lending standards through higher loan rejection rates during the downturn.

This paper also adds to the effort of building quantitative models of credit rationing. In a typical market equilibrium, price would equate supply and demand, and rationing should not exist. [Jaffee and Russell \[1976\]](#) and [Stiglitz and Weiss \[1981\]](#) show that rationing can happen in the presence of asymmetric information,

¹⁰In a related work, [Jiménez et al. \[2018\]](#) finds that firms with lower credit risk have a higher probability of being granted loan applications, and this pattern is stronger for less capitalized banks.

spawning further work on credit rationing based on an information-theoretic approach (e.g., [Bester \[1985\]](#), [Williamson \[1987\]](#)). In [Stiglitz and Weiss \[1981\]](#), the rationale for banks to reject some borrowers is negative adverse selection. The key insight is that a bank’s return does not always monotonically increase with the interest rate – in fact, it is an inverse U-shape. This paper shares this key insight, but for a different reason. The friction generating my result is costly state verification due to ex-post information asymmetry. More importantly, we explore the cyclical properties of credit rationing in a dynamic setting. In my model, credit rationing is closely linked with conditions of lenders’ balance sheets, which makes our framework suitable for studying financial crises.

The paper is structured as follows. Section [1.3](#) describes the partial equilibrium model in which banks set lending standards. Section [1.4](#) lays out the general equilibrium model. The model is taken to the data in Section [1.5](#). Section [1.6](#) conducts quantitative analysis, and Section [1.7](#) concludes.

1.3 Lending Standards in the Partial Equilibrium Model

In this section, I build a theoretical model to explain why credit rejection can be optimal for banks. I first discuss entrepreneurs’ heterogeneity and financing, and then analyze the relationship between banks’ expected revenue and borrowers’ risk in a partial equilibrium setting. The structure of the credit market follows [Bernanke et al. \[1999\]](#), in which there are a continuum of entrepreneurs (borrowers) and a representative bank (lender). The partial equilibrium analysis will be extended to

a dynamic general equilibrium framework in Section 3.

1.3.1 Firm Heterogeneity in Risk

Entrepreneurs, indexed by j , operate a firm that uses a linear technology to transform one unit of consumption goods at period t to $\varepsilon_{j,t+1}z_{t+1}R_{t+1}^k$ units of consumption goods at period $t + 1$, where $\varepsilon_{j,t+1}$ is the idiosyncratic productivity shock specific to entrepreneur j , z_{t+1} is the aggregate productivity shock, and R_{t+1}^k is the common return of capital, which will be determined in the general equilibrium.

Entrepreneurs are heterogeneous *ex-ante*. Each entrepreneur's idiosyncratic shock $\varepsilon_{j,t+1}$ is identically and independently drawn each period from a log-normal distribution with different mean and variance. Specifically,

$$\varepsilon_{j,t+1} \sim F(\varepsilon_{j,t+1}) = \ln \mathcal{N}(\mu_{j,t}, \sigma_{j,t})$$

We further assume that the parameter $\mu_{j,t}$ has the following structure: $\mu_{j,t} = -\sigma_{j,t}^2/2$. Therefore, the idiosyncratic shock has unit mean, which is the same across individuals ¹¹:

$$\mathbb{E}_t(\varepsilon_{j,t+1}) = \int_0^\infty (\varepsilon_{j,t+1}) dF(\varepsilon_{j,t+1}; \sigma_{j,t}) = 1$$

Entrepreneurs' heterogeneity can be characterized by their differing idiosyncratic

¹¹One extension here is to allow mean and variance to be correlated. For instance, we could assume that the parameter $\mu_{j,t}$ has the following structure:

$$\mu_{j,t} = \gamma \ln(\sigma_{j,t}) - \sigma_{j,t}^2/2$$

so that $\mathbb{E}_t(\varepsilon_{j,t+1}) = \sigma_{j,t}^\gamma$, where $\gamma \geq 0$ is a parameter governing the correlation between the mean and the variance of the idiosyncratic productivity, and they are positively correlated when $\gamma > 0$.

volatility $\sigma_{j,t}$, which we refer as the *risk* of the project. $\sigma_{j,t}$ follows a uniform distribution with c.d.f. H on the support $[a, b]$, i.e., $\sigma_{j,t} \sim \text{Uniform}[a, b]$. We assume that $\sigma_{j,t}$ is i.i.d. across entrepreneurs and over time. The aggregate shock z_{t+1} is an i.i.d. draw from a log-normal distribution $G(z_{t+1})$:

$$z_{t+1} \sim G(z_{t+1}) = \ln \mathcal{N}(-\sigma_z^2/2, \sigma_z)$$

where $\sigma_z > 0$ is the variance of the aggregate shock z_{t+1} .

1.3.2 Thresholds of Firm Default

At the end of period t , entrepreneur j has available net worth $N_{j,t}^E$. Let $Q_t k_{j,t+1}$ be entrepreneur j 's capital expenditure at the end of period t , where Q_t is the price of capital and $k_{j,t+1}$ is the quantity of capital. We will discuss how $k_{j,t+1}$ and $N_{j,t}^E$ are determined in what follows. To finance the difference between capital expenditure and net worth, the entrepreneur must borrow an amount $b_{j,t}$, given by

$$b_{j,t} = Q_t k_{j,t+1} - N_{j,t}^E \tag{1.1}$$

The amount $b_{j,t}$ is borrowed from the bank in the form of debt. Entrepreneur j promises to repay the bank at the face value $R_{t+1}^b b_{j,t}$ at time $t + 1$, where R_{t+1}^b is the gross interest rate.¹² After the realization of idiosyncratic and aggregate shocks at time $t + 1$, if the total value of the firm managed by entrepreneur j falls

¹²For the moment, I simply assume that the interest rate R_{t+1}^b is the same for all firms at any loan amount. I will discuss this assumption in detail in the next section.

below the promised amount of debt repayment, the entrepreneur defaults, which is characterized as follows:

$$\underbrace{\varepsilon_{j,t+1} z_{t+1} R_{t+1}^k Q_t k_{j,t+1}}_{\text{Firm Value}} \leq \underbrace{R_{t+1}^b b_{j,t+1}}_{\text{Loan repayment}} \quad (1.2)$$

I use $\bar{x}_{j,t+1}$ to denote firm j 's default threshold on total productivity, which is a product of the idiosyncratic and aggregate shock, given by

$$\varepsilon_{j,t+1} z_{t+1} \leq \bar{x}_{j,t+1} \equiv \frac{R_{t+1}^b b_{j,t}}{R_{t+1}^k Q_t k_{j,t+1}} \quad (1.3)$$

For realizations of $x_{j,t+1}$ above $\bar{x}_{j,t+1}$ the borrower pays $R_{t+1}^b b_{j,t}$ to the bank. For realizations below $\bar{x}_{j,t+1}$, the bank seizes the borrower's assets after paying a proportional monitoring cost μ_E . Firms' default can be attributed to an adverse idiosyncratic shock $\varepsilon_{j,t+1}$ to the firm, or an adverse aggregate shock z_{t+1} affecting all firms, or a combination of both. The two shocks have the same impact on an individual firm's ability to repay its debt, but they have different implications for the bank, which we will illustrate in the next section. Conditional on the aggregate shock z_{t+1} , the defaulting firms are those with realized idiosyncratic shocks lower than $\bar{x}_{j,t+1}/z_{t+1}$. In other words, $\bar{x}_{j,t+1}/z_{t+1}$ is the default threshold for the idiosyncratic shock.

1.3.3 Bank Revenue

I explore the relationship between banks' revenue and borrowers' risk. To fix ideas, suppose that the bank has extended a loan contract $\{R_t^b, b_t\}$ to an entrepreneur with risk σ_t at period t . For the moment let us take the loan contract as given. We will discuss the determination of the loan contract in what follows. I drop the individual subscript j to focus on the revenue from one particular loan. Given this loan contract, the bank's revenue from this contract is $\min\{R_t^b b_t, (1 - \mu_E)\varepsilon_{t+1} z_{t+1} R_{t+1}^k Q_t k_{t+1}\}$, which depends on the realized value of ε and z shocks, where $0 < \mu_E < 1$ is the monitoring cost. The bank's *expected* revenue, denoted by $V_t^B(\sigma_t; \mathcal{C})$, can be therefore expressed as

$$V_t^B(\sigma_t; \mathcal{C}) = \int_0^\infty \left(\underbrace{\int_{\frac{\bar{x}_{t+1}}{z}}^\infty R_t^b b_t dF_t(\varepsilon)}_{\text{Full Loan Repayment}} + (1 - \mu_E) \underbrace{\int_0^{\frac{\bar{x}_{t+1}}{z}} \varepsilon z R_{t+1}^k Q_t k_{t+1} dF_t(\varepsilon)}_{\text{The Value of Firm Under Default}} \right) dG(z) \quad (1.4)$$

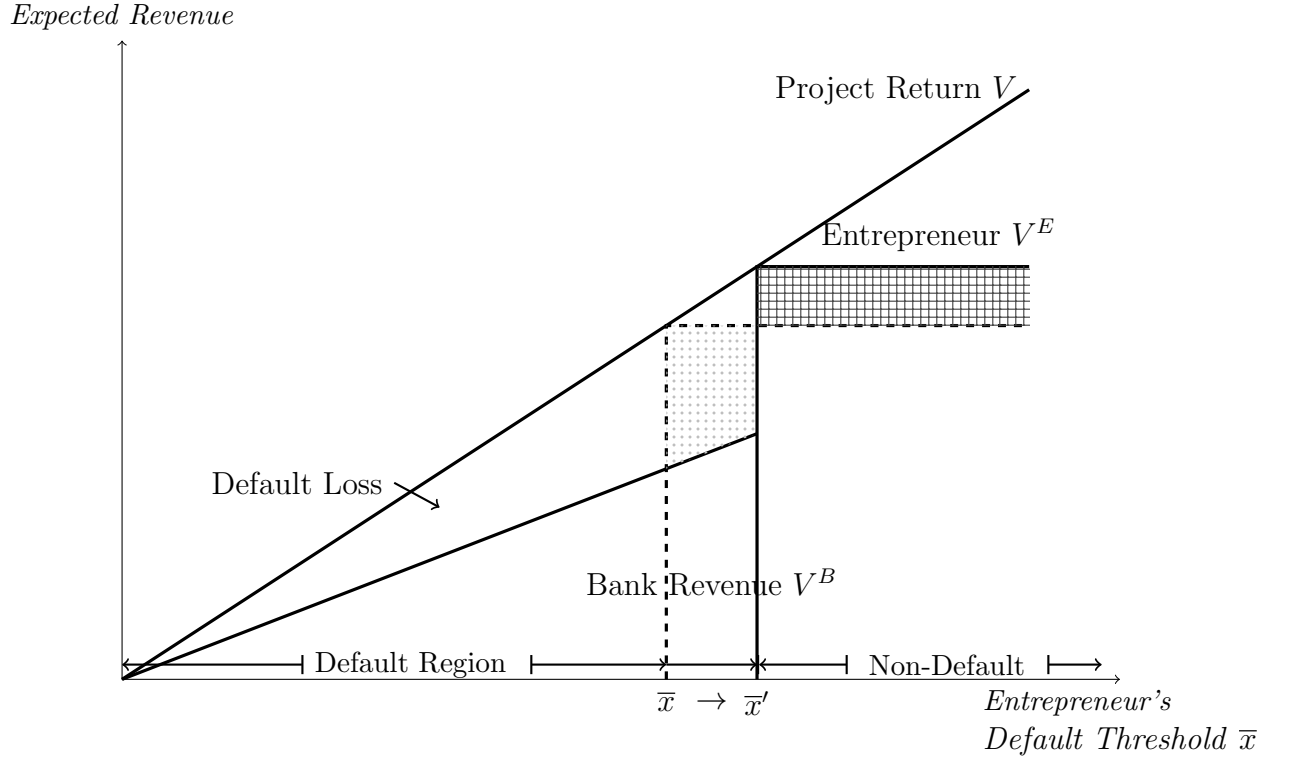
where \mathcal{C} denotes the contract and aggregate variables. The first term inside the parentheses of Equation (1.4) is the expected return when the entrepreneur does not default and makes full repayment. This happens when the realized value of the entrepreneur's idiosyncratic shock is greater than $\frac{\bar{x}_{t+1}}{z_{t+1}}$. The second term denotes the expected return when the entrepreneur defaults, in which case the bank liquidates the total value of the firm, subject to the monitoring cost. The idiosyncratic default threshold co-moves negatively with the realization of the aggregate shock z_{t+1} . The outside integral is taken over all possible realizations of aggregate shocks.

Using the definition of the firm default threshold in Equation (1.3), we can express the bank's expected revenue as a function of σ_t , contract variables $\{\bar{x}_{t+1}, k_{t+1}\}$ and aggregate variables $\{R_{t+1}^k, Q_{t+1}\}$, shown as follows:

$$V_t^B(\sigma_t; \mathcal{C}) = \mathcal{S}^B(\bar{x}_{t+1}, \sigma_t) R_{t+1}^k Q_{t+1} k_{t+1} \quad (1.5)$$

where $\mathcal{S}^B(\bar{x}_{t+1}, \sigma_t) \equiv \int_0^\infty ((1 - F_t(\frac{\bar{x}_{t+1}}{z_{t+1}}))^{\frac{\bar{x}_{t+1}}{z_{t+1}}} + (1 - \mu_E) \int_0^{\frac{\bar{x}_{t+1}}{z_{t+1}}} \varepsilon dF_t(\varepsilon)) z_{t+1} dG(z_{t+1})$ is the bank's expected share of the project. The bank's expected revenue is affected by both the firm default threshold \bar{x}_{t+1} and the entrepreneur's risk σ_t . For a given level of σ_t and k_{t+1} , changing the lending rate, and hence the firm's default threshold, has two effects on the bank's expected revenue, and they work in opposite directions. Figure 1.1 illustrates the effects of changing \bar{x}_{t+1} on the payoff to the bank and entrepreneur. A rise in \bar{x}_{t+1} increases the bank's expected revenue by raising the gross expected share that the bank receives in the case of non-default, illustrated by the shaded area. At the same time, a higher \bar{x}_{t+1} (a higher lending rate) increases entrepreneurs' probability of default and consequently raises deadweight loss, which lowers the bank's expected revenue, illustrated by the dotted area. The size difference between the shaded area and the dotted area varies with \bar{x}_{t+1} . When \bar{x}_{t+1} is small, the first positive effect exceeds the second negative effect so that the bank's expected revenue is increasing in \bar{x}_{t+1} . When \bar{x}_{t+1} is large, the second effect starts to dominate, in which case the bank's expected revenue starts to fall in \bar{x}_{t+1} .

Our first finding is that banks' revenue does not monotonically increase with the lending rate. In fact, bank's expected revenue is an inverse U-shaped function



Notes: This figure shows the effect of an increase in the lending rate on the payoff to entrepreneurs and banks. An increase in the lending rate 1-to-1 maps to an increase in firm's default threshold \bar{x}_{t+1} . The top-right shaded area is the marginal benefit arising from larger loan repayment. The bottom-left dotted area is the marginal cost arising from higher probability of default.

Figure 1.1: The Tradeoff of Increasing Lending Rate on Bank's Revenue

of the lending rate, as shown in Figure (1.2). This concave relationship implies that there exists a maximum for bank's expected revenue, and the bank cannot earn more than this maximum value for any lending rate. Result 1 summarizes this finding and characterizes this turning point:¹³

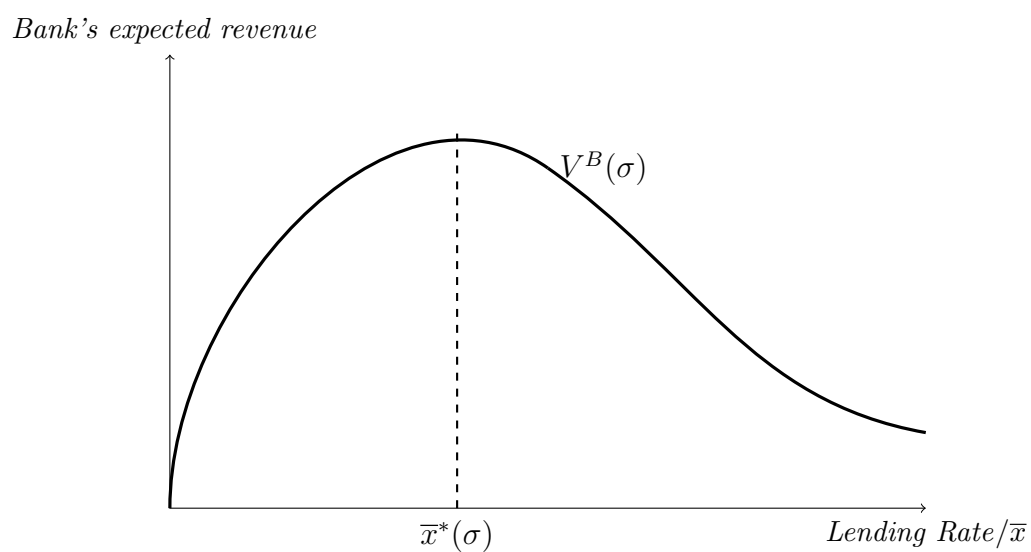
Result 1. *There exists a unique value $\bar{x}_{t+1}^*(\sigma_t)$ that maximizes the bank's expected revenue, i.e., $\bar{x}_{t+1}^*(\sigma_t) = \operatorname{argmax}_{\bar{x}_{t+1}} V_t^B(\sigma_t; \mathcal{C})$. $\bar{x}_{t+1}^*(\sigma_t)$ satisfies $\frac{\partial S^B(\bar{x}_{t+1}, \sigma_t)}{\partial \bar{x}_{t+1}} = 0$, and is defined by the following implicit function:*

$$\int_0^\infty \left[\left(1 - F_t\left(\frac{\bar{x}_{t+1}^*}{z_{t+1}}\right)\right) \right] dG(z_{t+1}) = \mu_E \int_0^\infty \left[f_t\left(\frac{\bar{x}_{t+1}^*}{z_{t+1}}\right) \frac{\bar{x}_{t+1}^*}{z_{t+1}} \right] dG(z_{t+1}) \quad (1.6)$$

The left-hand side of Equation (1.6) is the marginal benefit of raising \bar{x}_{t+1}^* : a rise in the default threshold maps one-to-one into an increase in the loan rate, which increases the total amount of revenue from non-defaulting firms. This marginal benefit is depicted as the shaded area in Figure 1.1. The right-hand side is the marginal cost of raising \bar{x}_{t+1}^* : it increases the share of defaulting firms and leads to more deadweight loss. The marginal cost is depicted as the dotted area in Figure 1.1. We denote as $\bar{V}_t^B(\sigma_t; \mathcal{C})$ the maximized value of banks' expected revenue, given by

$$\bar{V}_t^B(\sigma_t; \mathcal{C}) = \mathcal{S}^B(\bar{x}_{t+1}^*(\sigma_t), \sigma_t) R_{t+1}^k Q_t k_{t+1} \quad (1.7)$$

¹³As pointed out by Bernanke et al. [1999], the concavity requires some regularity conditions on the distribution function. In particular, the hazard function $x \rightarrow \frac{xf(x)}{1-F(x)}$ must be an increasing function. Standard distribution functions, such as the log-normal distribution as in this paper, satisfy this condition.



Notes: This figure shows that the bank's expected revenue is an inverse-U shaped function of the lending rate. For any given σ , there exists a unique value $\bar{x}^(\sigma)$ that yields a maximum expected revenue for the bank.*

Figure 1.2: Bank's Expected Revenue as a Function of Lending Rate

Moreover, there is a monotonic relationship between the borrower's risk and the bank's maximum expected revenue, summarized as follows:

Result 2. *The bank's maximum expected revenue $\bar{V}_t^B(\sigma_t; \mathcal{C})$ is decreasing in the entrepreneur's risk, i.e., $\partial \bar{V}_t^B(\sigma_t; \mathcal{C}) / \partial \sigma_t < 0$.*

The intuition is as follows. In debt contracts lenders' revenue is concave in the total project return. Consider an increase of risk in a mean-preserving manner. A higher volatility of the entrepreneur's return increases the left-tail risk, creating more default and hence lowering the lender's expected revenue. At the same time, the increase in right-tail risk does not benefit the lender because the loan repayment amount is predetermined. Overall, an increase in project volatility lowers the lender's expected revenue. Figure 1.3 plots the banks' expected revenue as a function of the borrower productivity default threshold for different borrower risk levels.

The inverse correlation between banks' maximum expected revenue and entrepreneurs' risk has implications for banks' participation in lending. As borrowers' risk rises, on the one hand, the bank's maximum expected revenue continues to fall. On the other hand, the bank's funding cost does not vary with borrowers' risk. At some point, risk will reach a point at which the bank's cost of funds exceeds the bank's highest possible revenue, in which case lending becomes absolutely unprofitable. The bank will stop lending to this borrower, and any other borrowers who are riskier. Formally, we characterize the bank's lending rule as follows:

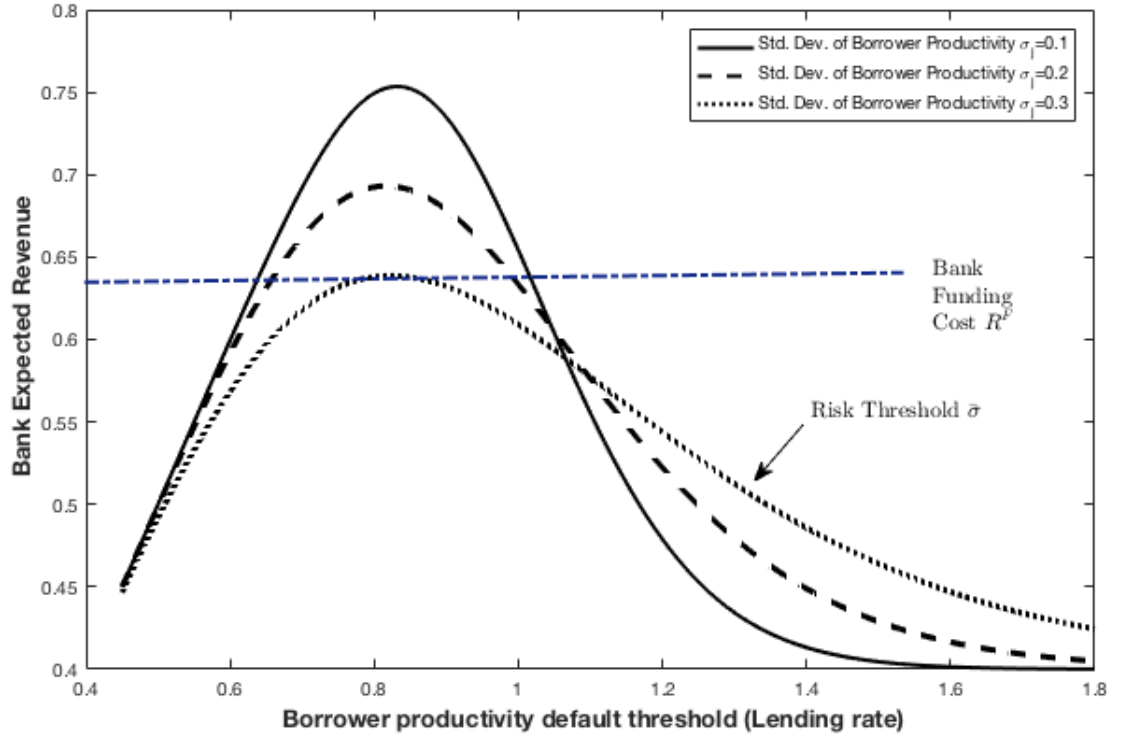
Result 3. *There exists a unique value $\bar{\sigma}_t$ at period t such that it is optimal for banks*

to reject borrowers whose risk is higher than $\bar{\sigma}_t$.¹⁴ $\bar{\sigma}_t$ is called the risk threshold, and satisfies

$$\bar{V}_t^B(\bar{\sigma}_t; \mathcal{C}) = \mathcal{S}^B(\bar{x}_{t+1}^*(\bar{\sigma}_t), \bar{\sigma}_t) R_{t+1}^k Q_t k_{t+1} = R_{t+1}^F b_t \quad (1.8)$$

The left-hand side of Equation (1.8) is the bank's maximum expected revenue from lending to a borrower with risk $\bar{\sigma}_t$. The right hand side is the bank's cost of funds R_{t+1}^F multiplied by loan amount b_t . Note that the risk threshold $\bar{\sigma}_t$ decreases in the bank's cost of funds R_{t+1}^F . That is, when banks' funding cost goes up, banks will lower the risk threshold so as not to lose money on their most risky borrowers. The determination of the risk threshold crucially depends on the bank's funding cost, which in turn is affected by its balance sheet condition. Therefore, we incorporate this mechanism into the general equilibrium model in what follows.

¹⁴It is worth noting that bank could continue to satisfy equation (1.8) by reducing b_t as borrowers' σ rises, in which case borrowers with $\sigma_t > \bar{\sigma}_t$ would be quantity (partial) rationed but not entirely rejected. We focus on the equilibrium with binary outcome (rejection or full funding) in this paper, and leave the case with partial rationing to future studies.



Notes: This figure shows bank's maximum expected revenue is decreasing in borrower's risk. The risk threshold $\bar{\sigma}$ is determined when the maximum expected revenue is equal to bank's funding costs.

Figure 1.3: Banks' Expected Revenue for Different Borrowers' Risk Levels

We interpret the risk threshold $\bar{\sigma}_t$ as a bank's *lending standards*. Such a cut-off lending rule is not uncommon in practice. In mortgage markets, for example, mortgage lenders require that borrowers must meet a sharply defined minimum credit score in order to qualify for a mortgage. Lenders deny credit to borrowers whose risk is above the threshold.¹⁵

1.4 The General Equilibrium Model

In this section, I embed the partial equilibrium results into a general equilibrium model. The main goal of this section is to investigate dynamic effects of changes in banks balance sheet conditions on lending standards and aggregate variables. All price and aggregate variables mentioned in the partial equilibrium setting will be endogenously determined in equilibrium.

The full model economy is composed of five types of agents: households, entrepreneurs (firms), banks, capital producers, and a mutual fund. On the financial side, the model structure is as follows. Households lend to the mutual fund in the form of deposits. The mutual fund uses the funds to lend to banks in the form of short-term debt. Banks combine this external funding and their own accumulated net worth to provide funding for entrepreneurs in the form of debt. Entrepreneurs combine the bank loans and their own accumulated net worth to start firms and produce goods. The main market frictions, as we mentioned in the section above, are the existence of CSV between banks and firms and also between banks and the

¹⁵[Laufer and Paciorek \[2016\]](#) document similar lending patterns in US mortgage markets. They find that lenders set minimum thresholds for consumers' FICO credit scores, and adjust the threshold values based on economic and financial conditions.

mutual fund. This new upper layer of CSV enables us to study the balance sheet dynamics of banks.

Similar to [Gertler and Kiyotaki \[2010\]](#), we adopt the island economy setup by assuming that banks and entrepreneurs are segmented across a continuum of islands. There is a representative bank in each island, providing financial intermediation services to the entrepreneurs located in the same island. The mutual fund operates economy-wide and diversifies perfectly across islands. Labor and consumption goods are also perfectly mobile.

We now analyze the behavior of each type of agent, and define the equilibrium.

1.4.1 Entrepreneurs

As we mentioned in [Section 1.3](#), an entrepreneur operates a firm with a linear production technology $Q_t K_{j,t+1} \rightarrow \varepsilon_{j,t+1} z_{t+1} R_{t+1}^k Q_t K_{j,t+1}$. In this section, I formally present the entrepreneur's problem, describe the sources of shocks and determine the common return of capital R_{t+1}^k .

1.4.1.1 Technology

There is a continuum of entrepreneurs of measure one on each island, and each entrepreneur manages a firm indexed by j .¹⁶ Firms are perfectly competitive. They are the consumption goods producers in this economy. All firms operate the same constant returns to scale Cobb-Douglas production function using effective capital

¹⁶Since each island is identical ex-ante, we will suppress the island index to have a compact notation.

and labor:

$$Y_{j,t+1} = \tilde{k}_{j,t+1}^\alpha (A_{t+1} L_{j,t+1})^{1-\alpha} \quad (1.9)$$

where A_{t+1} is the labor-augmented total factor productivity (TFP), and $Y_{j,t+1}$, $\tilde{k}_{j,t+1}$, and $L_{j,t+1}$ are the individual firm's output, effective capital, and labor demand, respectively.

At time t , firm j purchases raw capital, denoted by $k_{j,t+1}$, for use at $t+1$. The actual quantity of capital that can be used for production at time $t+1$, denoted by the effective capital $\tilde{k}_{j,t+1}$, is random. In particular, for $k_{j,t+1}$ units of capital firm j purchases at time t , the total amount of effective capital ready for production at time $t+1$ is $\tilde{k}_{j,t+1} = x_{j,t+1} k_{j,t+1}$, where $x_{j,t+1}$ is a composition of two shocks to firm j 's capital stock. Specifically, $x_{j,t+1}$ comprises a firm-specific shock component $\varepsilon_{j,t+1}$ and an island-wide shock component z_{t+1} , and $x_{j,t+1} = \varepsilon_{j,t+1} z_{t+1}$.¹⁷ The firm has $(1 - \delta)\tilde{k}_{j,t+1}$ units of undepreciated capital available for resale after production.¹⁸

1.4.1.2 Labor Choice and Firm Value

At time $t+1$, firm j chooses labor after the realization of the idiosyncratic and the island-wide shocks, given the capital stock purchased from last period. Hence, we can solve the firm's labor choice through a static profit maximization problem.

¹⁷In contrast to the partial equilibrium model where z_{t+1} denotes the aggregate shock since there is only one representative bank, now we denote z_{t+1} as an island-wide shock that will only affect the bank in this island. In essence, the z_{t+1} shock captures the undiversifiable risk in a bank's loan portfolio.

¹⁸The capital quality shocks capture the risk in operating actual business ventures. Similar shocks are seen in [Gertler and Kiyotaki \[2010\]](#), [Christiano et al. \[2014\]](#), and [Gourio \[2013\]](#). [Christiano et al. \[2014\]](#), for example, explains that "in the hands of some entrepreneurs, a given amount of raw capital (e.g., metal, glass, and plastic) is a great success (e.g., the Apple iPad or the Blackberry cell phone), and in other cases, it is less successful (e.g., the NeXT computer or the Blackberry Playbook)."

Given the capital stock, the aggregate wage and the realized shocks, firm j chooses labor $L_{j,t+1}$ to maximize its profit $\Pi_{j,t+1}$:

$$\max_{L_{j,t+1}} \Pi_{j,t+1} = \tilde{k}_{j,t+1}^\alpha (A_{t+1} L_{j,t+1})^{1-\alpha} - W_{t+1} L_{j,t+1} \quad (1.10)$$

Profit maximization yields the optimal labor demand as a function of capital, given by

$$L_{j,t+1} = \tilde{k}_{j,t+1} \left(\frac{A_{t+1}^{1-\alpha} (1-\alpha)}{W_{t+1}} \right)^{1/\alpha} \quad (1.11)$$

Let $V_{j,t+1}^E$ denote firm j 's total value at period $t+1$ after production (but before making loan repayments), where the superscript E denotes entrepreneur. The entrepreneur who experienced shock $x_{j,t+1}$ is left with $(1-\delta)x_{j,t+1}k_{j,t+1}$ units of capital after depreciation. This capital is sold in competitive markets at the price Q_{t+1} . Hence firm j 's value is the sum of maximized profit plus the value of undepreciated effective capital: $V_{j,t+1}^E = \Pi_{j,t+1} + (1-\delta)Q_{t+1}\tilde{k}_{j,t+1}$. Substituting for the optimal labor demand and rearranging gives firm j 's value as

$$V_{j,t+1}^E = \left[\alpha \left(\frac{A_{t+1}(1-\alpha)}{W_{t+1}} \right)^{(1-\alpha)/\alpha} + (1-\delta)Q_{t+1} \right] x_{j,t+1}k_{j,t+1} \quad (1.12)$$

Let R_{t+1}^k denote the return on effective capital, which is common across all entrepreneurs:

$$R_{t+1}^k \equiv \frac{\alpha \left(\frac{A_{t+1}(1-\alpha)}{W_{t+1}} \right)^{(1-\alpha)/\alpha} + (1-\delta)Q_{t+1}}{Q_t} \quad (1.13)$$

Therefore, we write firm j 's value in compact notation as $V_{j,t+1}^E = x_{j,t+1} R_{t+1}^k Q_t k_{j,t+1} = \varepsilon_{j,t+1} z_{t+1} R_{t+1}^k Q_t k_{j,t+1}$. That is, firm j 's individual return on raw capital is the product of the idiosyncratic shock, the island-specific shock and the common return on effective capital: $\varepsilon_{j,t+1} z_{t+1} R_{t+1}^k$.

1.4.2 Banks

In each island there exists a representative bank. Each entrepreneur applies for a business loan from the bank located in the same island. Banks apply the lending rule as determined in section 1.3 to decide on whether to reject borrowers. Given this lending standard, I describe the determination of bank's loan portfolio, bank's financing problem and default decisions in this subsection.

When banks decide on lending standards they have full information on the distribution of borrowers' risk, which is the time-invariant distribution $H(\sigma_t)$. In other words, the determination of $\bar{\sigma}_t$ does not depend on any individual borrower's risk. I assume the following information structure in implementing the lending standards.

Assumption 1. *Banks can only detect whether a given applicant's risk σ_j is above or below its chosen risk threshold; banks cannot observe the exact value of the borrower's risk σ_j .*

This assumption implies that all approved borrowers appear identical to the bank. As a result, all qualified borrowers receive the same credit contract.¹⁹ En-

¹⁹This assumption reduces the computational burden of the model without altering the main insight. Without this imperfect screening assumption, banks would offer a continuum of lending

entrepreneurs do not have information about their project's risk and the cost of applying for a bank loan is negligible. Hence, each entrepreneur will apply for a loan and there is no signaling about their risk type in equilibrium. For borrowers that are approved for credit, we can therefore drop the individual entrepreneur's subscript j from $b_{j,t}$ and $k_{j,t+1}$ so that $b_t = b_{j,t}$ and $k_{t+1} = k_{j,t+1}$.²⁰ Entrepreneurs that are denied credit have to run their businesses using their own net worth.

1.4.2.1 Loan Portfolio and Bank Default

The bank provides b_t in loans to each qualified entrepreneur. The total size of the bank's loan portfolio at time t , B_t , is given by

$$B_t = \int_a^{\bar{\sigma}_t} b_t dH(\sigma_{j,t}) \quad (1.14)$$

The integral over risk type is conditional on entrepreneurs being within the risk threshold, i.e., $dH(\sigma_{j,t} | \sigma_{j,t} < \bar{\sigma}_t)$, because lending happens after the bank's acceptance/rejection decision. The bank has equity N_t^B at the beginning of period t . To finance the difference between its assets and equity, the bank must borrow from the mutual fund. Similar to firms' financing problem, we assume banks borrow from the mutual fund in the form of debt. At period $t + 1$, banks will attempt to repay the debt at its face value $R_{t+1}^d D_t$. R_{t+1}^d is banks' borrowing rate. Note that since

rates based on the exact type of borrower they observe. In that case, the mechanism of this paper would still work, since banks would still impose a maximum risk threshold that depends on the bank's cost of funds.

²⁰This paper focuses on borrowers' heterogeneity along the dimension of risk. Therefore, I assume that entrepreneurs pool their net worth at the end of each period so that their starting net worth is the same. In future research, it is worth exploring borrowers' heterogeneity in the dimension of net worth.

bank debt is risky, there is a positive spread between banks' borrowing rate and the risk-free rate, i.e., $R_{t+1}^d - R_t > 0$. The bank's balance sheet at the end of period t is

$$B_t = N_t^B + D_t \quad (1.15)$$

Conditional on the realization of z_{t+1} and on being qualified, the expected value to the bank of an individual loan extended at period t to entrepreneur j is

$$\int_a^{\bar{\sigma}_t} \mathcal{S}^B(\bar{x}_{t+1}, z_{t+1}, \sigma_{j,t}) dH(\sigma_{j,t} | \sigma_{j,t} < \bar{\sigma}_t) R_{t+1}^k Q_t k_{t+1}$$

Summing over all approved borrowers, we obtain the total value of the bank's loan portfolio at period $t + 1$, denoted by V_{t+1}^B , given by

$$V_{t+1}^B = \left[\int_a^{\bar{\sigma}_t} \mathcal{S}^B(\bar{x}_{t+1}, z_{t+1}, \sigma_{j,t}) dH(\sigma_{j,t} | \sigma_{j,t} < \bar{\sigma}_t) \right] R_{t+1}^k Q_t K_{t+1}^f \quad (1.16)$$

where $K_{t+1}^f = \int_a^{\bar{\sigma}_t} k_{t+1} dH(\sigma_{j,t})$ is the total capital purchased by bank-financed entrepreneurs.

V_{t+1}^B is increasing in the aggregate shock z_{t+1} . When the realized aggregate shock is sufficiently low, the bank's total value of assets falls below the total value of its liabilities, in which case the bank defaults. Due to the monotonic relationship between z_{t+1} and the bank's revenue, there exists a unique value, z_{t+1}^* , below which the bank is insolvent and subsequently will default. Banks' threshold, z_{t+1}^* ,

is uniquely determined by

$$\left[\int_a^{\bar{\sigma}_t} \mathcal{S}^B(\bar{x}_{t+1}, z_{t+1}^*; \sigma_{j,t}) dH(\sigma_{j,t} | \sigma_{j,t} < \bar{\sigma}_t) \right] R_{t+1}^k Q_t K_{t+1}^f = R_{t+1}^d D_t \quad (1.17)$$

The right hand side of Equation (1.17) is the total liability at period $t+1$: the bank needs to pay its debt D_t to the mutual fund at a gross interest rate R_{t+1}^d .

1.4.3 The Mutual Fund

The mutual fund takes deposits from the household and lends to a continuum of banks. The mutual fund can diversify over the island-specific shocks, so that its deposits pay the risk-free interest rate R_t to the household. The mutual fund's participation constraint is

$$\underbrace{(1 - G(z_{t+1}^*)) R_{t+1}^d D_t}_{\text{Non-defaulting Banks}} + \underbrace{(1 - \mu_B) \int_0^{z_{t+1}^*} \int_a^{\bar{\sigma}_t} \mathcal{S}^B(\bar{x}_{t+1}, z, \sigma) dH(\sigma | \sigma < \bar{\sigma}_t) dG(z) R_{t+1}^k Q_t K_{t+1}^f}_{\text{Defaulting Banks}} \geq R_t D_t \quad (1.18)$$

where $0 < \mu_B < 1$ is the monitoring cost when banks default. The first term of the above expression is banks' full payment when not in default, i.e., the face value of bank's debt $R_{t+1}^d D_t$. When banks default, however, banks' debt holders claim the banks' assets, net of the proportional monitoring cost μ_B , denoted by the second term. Equation (1.18) states that the total revenue from lending to banks must be at least as large as the cost of paying household deposits at the risk-free interest rate R_t .

1.4.4 The Financial Contracts

We can now specify the joint optimization problem and determine the financial contract. Agents choose $\{\bar{x}_{t+1}, b_t\}$ to maximize the expected profit of entrepreneurs who are approved for credit, subject to the participation constraints of banks and the mutual fund. In other words, banks choose the risk threshold according to (1.8), and then $\{\bar{x}_{t+1}, b_t\}$ are chosen to maximize the expected profit of entrepreneurs who fall within the risk threshold.

An entrepreneur's share of the return from the project is $\max \{\varepsilon_{t+1} z_{t+1} R_{t+1}^k Q_t k_{t+1} - R_t^b b_t, 0\}$.²¹ The entrepreneur's expected profit conditional on being approved for credit is

$$\begin{aligned} & \mathbb{E}_t \int_a^{\bar{\sigma}_t} \int_0^\infty \int_0^\infty \max \{ \varepsilon_{t+1} z_{t+1} R_{t+1}^k Q_t k_{t+1} - R_t^b b_t, 0 \} dF(\varepsilon_{t+1}) dG(z_{t+1}) dH(\sigma_{j,t} | \sigma_{j,t} < \bar{\sigma}_t) \\ &= \mathbb{E}_t \left[\int_a^{\bar{\sigma}_t} \mathcal{S}^E(\bar{x}_{t+1}, \sigma_{j,t}) dH(\sigma_{j,t} | \sigma_{j,t} < \bar{\sigma}_t) R_{t+1}^k Q_t k_{t+1} \right] \end{aligned} \quad (1.19)$$

where $\mathcal{S}^E(\bar{x}_{t+1}, \sigma_{j,t}) \equiv \int_0^\infty \int_{\frac{\bar{x}_{t+1}}{z_{t+1}}}^\infty (\varepsilon - \frac{\bar{x}_{t+1}}{z_{t+1}}) z_{t+1} dF(\varepsilon; \sigma_{j,t}) dG(z_{t+1})$ denotes the expected share of the project return owned by an entrepreneur, conditional on particular realized values for risk $\sigma_{j,t}$.

²¹There is also a participation constraint for entrepreneurs, which requires the expected profit from taking the financial contract should be at least as large as investing entrepreneurs' net worth earning a risk-free interest rate:

$$\mathbb{E}_t \left[\int_a^{\bar{\sigma}_t} \mathcal{S}^E(\bar{x}_{t+1}, \sigma_{j,t}) dH(\sigma_{j,t} | \sigma_{j,t} < \bar{\sigma}_t) \right] R_{t+1}^k Q_t k_{t+1} \geq R_t(Q_t k_{t+1} - b_t)$$

We assign a Lagrange multiplier to this constraint when solving the model and find that the multiplier is non-zero in our calibration, which implies that constraint is not binding and entrepreneurs always participate in the financial contract.

The bank's participation constraint is that the expected revenue from total loans, net of the payment to the bank's debt holders, is at least as large as the opportunity cost of bank equity, which would otherwise earn the risk-free interest rate. The bank's expected profit from lending to an entrepreneur is

$$\begin{aligned} & \max \left\{ \int_a^{\bar{\sigma}_t} \mathcal{S}^B(\bar{x}_{t+1}, z_{t+1}, \sigma_{j,t}) dH(\sigma_{j,t} | \sigma_{j,t} < \bar{\sigma}_t) R_{t+1}^k Q_t k_{t+1} - R_{t+1}^F b_t, 0 \right\} \\ &= \int_{z_{t+1}^*}^{\infty} \int_a^{\bar{\sigma}_t} \mathcal{S}^B(\bar{x}_{t+1}, z_{t+1}, \sigma_{j,t}) dH(\sigma_{j,t} | \sigma_{j,t} < \bar{\sigma}_t) dG(z_{t+1}) R_{t+1}^k Q_t k_{t+1} - (1 - G(z_{t+1}^*)) R_{t+1}^F b_t \end{aligned} \quad (1.20)$$

The outside integral sums over realizations of the aggregate shock z_{t+1} that are above z_{t+1}^* , in which case the bank does not default. The bank's total expected profit from the *loan portfolio* is an aggregation over profits of individual loans in (1.20), given by

$$\left[\int_{z_{t+1}^*}^{\infty} \int_a^{\bar{\sigma}_t} \mathcal{S}^B(\bar{x}_{t+1}, z_{t+1}, \sigma_{j,t}) dH(\sigma_{j,t} | \sigma_{j,t} < \bar{\sigma}_t) dG(z_{t+1}) \right] R_{t+1}^k Q_t K_{t+1}^f - (1 - G(z_{t+1}^*)) R_{t+1}^F B_t \quad (1.21)$$

where R_{t+1}^F is the bank's funding cost. Note that R_{t+1}^F is an implicit cost, which is not the interest rate on any loan contract.²² It is determined by the participation constraint of banks' shareholders:

$$(1 - G(z_{t+1}^*)) (R_{t+1}^F B_t - R_{t+1}^d D_t) = R_t N_t^B \quad (1.22)$$

²²Note that R^F , the bank's cost of funds, is not the same as R^d , which is the interest rate on banks' debt. This is because that banks' funding is composed of both internal net worth and external debt, which have different costs. The cost of internal net worth is the opportunity cost of investing at the risk-free interest rate.

Combining the above two equations, we can write the bank's zero profit condition as

$$\left[\int_{z_{t+1}^*}^{\infty} \int_a^{\bar{\sigma}_t} \mathcal{S}^B(\bar{x}_{t+1}, z_{t+1}, \sigma_{j,t}) dH(\sigma_{j,t} | \sigma_{j,t} < \bar{\sigma}_t) dG(z_{t+1}) \right] R_{t+1}^k Q_t K_{t+1}^f - (1 - G(z_{t+1}^*)) R_{t+1}^d D_t = R_t N_t^B \quad (1.23)$$

where the bank default threshold z_{t+1}^* is defined in Equation (1.17). The optimal financial contract is a set $\{\bar{x}_{t+1}, b_t\}$ such that it maximizes entrepreneurs' profit in (1.19) subject to banks' zero profit condition in (1.23), where R_{t+1}^d is determined by the mutual fund's participation constraint in (1.18) and the threshold value of banks' default z_{t+1}^* is determined in (1.17), conditional on banks' risk threshold $\bar{\sigma}_t$ in (1.8).

The optimality conditions of the financial contract are documented in Appendix (A.2).

1.4.5 Households

Households' problem is standard. There is a unit measure of identical households in this economy. The representative household chooses consumption C_t , labor supply L_t^H and deposit saving D_{t+1}^H to maximize the expected lifetime utility given by

$$\max \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t U(C_t, L_t^H) \right]$$

subject to the budget constraint

$$C_t + D_{t+1}^H/R_t = W_t L_t^H + D_t^H + \Pi_t \quad (1.24)$$

where $U(\cdot)$ is the period felicity utility, R_t the risk-free interest rate, W_t the wage rate, and Π_t the total profits and dividends transferred from the entrepreneurs and banks. The first order conditions with respect to consumption, labor, and savings yield the following optimality conditions:

$$\mathbb{E}_t(M_{t+1}R_t) = 1 \quad (1.25)$$

$$U_C(C_t, L_t^H)W_t = U_L(C_t, L_t^H) \quad (1.26)$$

$$M_{t+1} = \beta U_C(C_{t+1}, L_{t+1}^H)/U_C(C_t, L_t^H) \quad (1.27)$$

1.4.6 New Capital Producers

The perfectly competitive capital producer transforms final consumption goods into capital. The production of new capital is subject to adjustment costs. In particular, capital producers take $(1 + S(\frac{I_t}{I_{t-1}}))I_t$ consumption goods and transform them into I_t investment goods that are sold at price Q_t . $S(\frac{I_t}{I_{t-1}})$ is the convex adjustment cost, which satisfies $S(1) = S'(1) = 0$ and $S''(1) \equiv \zeta > 0$. The capital producer's objective function is

$$\max_{I_t} \sum_{t=0}^{\infty} \beta^t M_{0,t} [Q_t I_t - (1 + S(\frac{I_t}{I_{t-1}}))I_t] \quad (1.28)$$

where $M_{0,t}$ is the stochastic discount factor. The first order condition of the capital producer's optimization problem is

$$Q_t = 1 + S\left(\frac{I_t}{I_{t-1}}\right) + \frac{I_t}{I_{t-1}} S'\left(\frac{I_t}{I_{t-1}}\right) - \beta E_t \left[M_{t,t+1} \left(\frac{I_{t+1}}{I_t}\right)^2 S'\left(\frac{I_{t+1}}{I_t}\right) \right] \quad (1.29)$$

1.4.7 Equity Dynamics

By the law of large numbers, the aggregate profit of all firms at the end of period t , V_t^E , is the sum of realized profits of bank-financed firms and the profits of self-financed firms:

$$\begin{aligned} V_t^E = & \int_a^{\bar{\sigma}_{t-1}} \int_0^\infty \mathcal{S}^E(\bar{x}_t, z, \sigma) dG(z) dH(\sigma) R_t^k Q_{t-1} K_t^f \\ & + \int_{\bar{\sigma}_{t-1}}^b \int_0^\infty \int_0^\infty \varepsilon z dF(\varepsilon; \sigma) dG(z) dH(\sigma) R_t^k Q_{t-1} K_t^{sf} \end{aligned} \quad (1.30)$$

where K_t^{sf} is the total capital purchased by self-financed firms using their own equity:

$$\begin{aligned} Q_{t-1} K_t^f &= B_t + \int_a^{\bar{\sigma}_{t-1}} N_t^E dH(\sigma_{j,t-1}) \\ Q_{t-1} K_t^{sf} &= \int_{\bar{\sigma}_{t-1}}^b N_t^E dH(\sigma_{j,t-1}) \end{aligned}$$

The evolution of net worth in this paper closely follows [Christiano et al. \[2014\]](#). Entrepreneurs' net worth in period $t+1$ is comprised of retained earnings and labor income. Entrepreneurs are assumed to supply L^E units of labor inelastically each period. This is a standard assumption in the literature to ensure equity always being positive. I assume that entrepreneurs have unit measure and that they pool

resources at the end of each period. I also assume that entrepreneurs retain a fraction γ^E of earnings and rebate the remaining share $(1 - \gamma^E)$ as a dividend to households. Therefore, the net worth of an entrepreneur at the start of period $t + 1$ (or equivalently the end of period t) evolves as follows:

$$N_{t+1}^E = \gamma^E V_t^E + W_t L^E$$

Profits rebated to households as dividends are:

$$\Pi_t^E = (1 - \gamma^E) V_t^E$$

Similarly, banks' realized profit at the end of period t is:

$$V_t^B = \int_a^{\bar{\sigma}_{t-1}} \int_{z_t^*}^{\infty} \mathcal{S}^B(\bar{x}_t, z, \sigma) dG(z) dH(\sigma | \sigma < \bar{\sigma}_t) R_t^k K_t^f - (1 - G(z_t^*)) R_t^d D_{t-1}$$

At the end of each period, banks pay a fraction $(1 - \gamma^B)$ of their profits as dividends to the household. The remaining fraction γ^B is injected into new equity as retained earnings. The banker supplies L^B units of labor inelastically. Banks' equity at the start of period $t + 1$ is the sum of retained earnings and bankers' labor income:

$$N_{t+1}^B = \gamma^B V_t^B + W_t L^B; \quad \Pi_t^B = (1 - \gamma^B) V_t^B$$

1.4.8 Market Clearing

In the labor market, aggregate labor supply is the sum of labor supply from households, entrepreneurs, and bankers. Aggregate labor demand is the sum of labor demand from firms receiving external financing plus labor demand from self-financed firms. Therefore, the labor market clearing condition is: $L_t = L_t^H + L^E + L^B$. The total capital is $K_t = K_t^f + K_t^{sf}$. The funds market clears when household deposits equal the mutual fund's total lending, i.e., $D_t = D_t^H$. Lastly, the economy-wide resource constraint is given by

$$\begin{aligned}
Y_t = & C_t + I_t + \Theta(I_t) + \mu_E \int_{z_t^*}^{\infty} \int_a^{\bar{\sigma}_{t-1}} \int_0^{\bar{x}/z} \varepsilon z dF(\varepsilon) dG(z) dH(\sigma | \sigma < \bar{\sigma}_{t-1}) R_t^k K_t^f \\
& + \mu_B \int_0^{z_t^*} \int_a^{\bar{\sigma}_{t-1}} \mathcal{S}^B(\bar{x}_t, z, \sigma) dG(z) dH(\sigma | \sigma < \bar{\sigma}_{t-1}) R_t^k K_t^f
\end{aligned} \tag{1.31}$$

$\Theta(I_t)$ is the capital adjustment cost. The last two terms on the right-hand side of Equation (1.31) reflect the resources used for monitoring defaulting firms and banks, respectively.

The complete list of optimality conditions is documented in Appendix (A.2).

1.5 Calibration

1.5.1 Choosing Parameters

The model is calibrated to the United States economy at a quarterly frequency. The calibration strategy is designed to ensure that steady state conditions of the

model match with their data counterparts. The calibrated parameters can be divided into two groups. The parameters in the first group (the top panel in Table 1.1) either have direct data counterparts or are standard in the literature. The parameters in the second group (the bottom panel in Table 1.1) are particular to this model, and usually do not have direct data counterparts. For this set of parameters, the values are jointly calibrated so that the model matches a set of relevant moments from the data.

Parameter	Value	Description	Source/Target
β	0.996	Discount factor	the risk-free rate $R = 1.6\%$
α	0.35	Capital share in firm production	Standard RBC
δ	0.025	Capital depreciation rate	Standard RBC
φ	1	Inverse labor supply elasticity	Comin and Gertler [2006]
ζ	0.5	Investment adjustment costs	Jermann and Quadrini [2012]
h	0.7	Consumption habits	Smets and Wouters [2007]
ψ_L	0.3	Disutility weight on labor	labor supply at 1/3 units
L^E	0.01	Entrepreneurs' labor supply	Christiano et al. [2014]
L^B	0.01	Bankers' labor supply	Christiano et al. [2014]

Table 1.1: Calibration I: Standard Parameters

Household preferences are given by

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left(\ln(C_{t+s} - hC_{t+s-1}) - \frac{\psi_L}{1+\varphi} (L_{t+s}^H)^{1+\varphi} \right)$$

with $0 < \beta < 1, 0 < h < 1$ and $\psi_L, \varphi > 0$. This preference specification is standard in the literature. It allows for habit formation to capture consumption dynamics (e.g., [Gertler and Karadi \[2011\]](#)). The capital adjustment cost is assumed to take the functional form $S(x) = \frac{1}{2}\zeta(x-1)^2$. In equilibrium, ζ is the inverse elasticity of investment with respect to the price of capital.

Parameter	Value	Description	Target Moment	Model	Data
b	0.65	upper bound of σ_ε dist.	Loan approval rate	82.4%	78%
μ_B	0.25	bank's monitoring cost	Bank funding credit spread	3.6%	2.74%
μ_E	0.65	firm's monitoring cost	LGD of Bank Loan	50.2%	49.3%
a	0.05	lower bound of σ_ε dist.	Firms' Prob. Default	6.6%	5.37%
γ^E	0.88	% of firm's retained earnings	Firm asset-to-equity ratio	2.3	2
γ^B	0.64	% of bank's retained earnings	Bank asset-to-equity ratio	13.7	13.6
σ_z	0.12	standard deviation of z shock	Banks' Prob. Default	0.92%	0.87%

Notes: Moments are annualized. Model moments are evaluated at the steady state. See Appendix A.4 for the definition of moments.

Table 1.2: Calibration II: Internally Calibrated Parameters and Moments

The first group of parameters are chosen as follows. The household discount rate is set to $\beta = 0.996$, implying an approximately 1.6% annual risk-free interest rate. The share of capital in output production is $\alpha = 0.35$, and the capital depreciation rate δ is equal to 0.025, which are both standard values in the Real Business Cycle (RBC) literature. I set the inverse of the Frisch elasticity of labor supply φ at unity, which represents an intermediate value for the range of estimates across micro and macro literature. It is also in line with other macroeconomic studies (e.g., [Comin and Gertler \[2006\]](#)). The investment adjustment cost parameter ζ is set to 0.5 as in [Jermann and Quadrini \[2012\]](#). The consumption habit parameter h is estimated to be 0.71 in [Smets and Wouters \[2007\]](#) and 0.65 in [Christiano et al. \[2005\]](#). I set h equal to 0.7. I calibrate ψ_L so that households' labor supply L^H is equal to 1/3 in the steady state, capturing a standard eight hour work day. The labor supply of entrepreneurs and bankers plays a minimal role in the consumption goods production process, but it is important to include them since it ensures both agents having non-zero initial net worth in case of default. Following [Christiano et al. \[2014\]](#), I assign a small value, 0.01, to both parameters.

Parameters in the second group do not have direct data counterparts and hence are not individually identifiable. Instead, there are a set of model objects depending on these parameters in the steady state. The calibration strategy is to choose these parameters jointly so that the values of model objects are as close as possible to the values of their data counterparts. The group of data moments and their sources are described as follows. Data on loan approval rates is from the Small Business Credit Survey conducted by the New York Federal Reserve Bank and the National

Federation of Independent Business(NFIB). The average loan approval rate since 2012 is 78%.²³ The leverage ratio of the corporate business sector, i.e., the asset-to-equity ratio, is close to 2 in the aggregate following [Gertler and Kiyotaki \[2010\]](#). I obtain the leverage ratio of the financial sector using aggregate bank balance sheet data from the Federal Deposit Insurance Corporation (FDIC). The average leverage ratio, defined as the asset-to-equity ratio, of the U.S. banking sector is 13.7 for all FDIC-insured commercial banks and savings institutions from 1990 to 2016.²⁴ The corporate probability of default (PD) is 5.37% in the US as documented in [De Fiore and Uhlig \[2011\]](#). Using data from the FDIC bank default database, I calculate the average PD of financial institutions from 1930 to 2016 as 0.8%.²⁵ The average financial funding spread, as measured by the spread between the average US financial bond yield and the 3-month Treasury bill yield from 1990 to 2016, is 2.74%.²⁶ To assess the real cost of corporate borrowing, I follow the literature and use the average corporate funding spread, as measured by the spread between the Baa corporate bond yield and the 3-month Treasury bill yield, which is 4.22% for the sample period 1990 to 2016.²⁷ Loss given default (LGD) is a widely used measure of credit risk. It measures the share of loans that are lost if a borrower

²³The Small Business Credit Survey (SBCS) (<https://www.newyorkfed.org/smallbusiness>) is an annual survey of small firms reporting on financing needs and outcomes. In 2015, the SBCS yielded 5,420 responses from businesses in 26 states. The Survey starts from 2010, and I take the average from 2012 to 2017, which yields the number 78%.

²⁴The bank asset-to-equity ratio is calculated by dividing the series of Total Assets by the series of Total Equity Capital.

²⁵The dataset on failed banks can be downloaded from <https://www.fdic.gov/bank/individual/failed/>.

²⁶ The financial bond yield is measured by the series of US Credit Bond Yield (Finance) from Citigroup Global Markets.

²⁷ The data can be downloaded from Bloomberg. The name of the series is “Moody’s Seasoned Baa Corporate Bond”.

defaults. According to a recent study by FDIC, the average LGD for Commercial and Industrial (C&I) loans is 49.3% from 2008 to 2013 (Shibut and Singer [2015]).

A summary on how model objects are mapped to the data is given in Table (1.2). The model overall provides a good fit of the targeted moments.

1.5.2 Parameter Identification

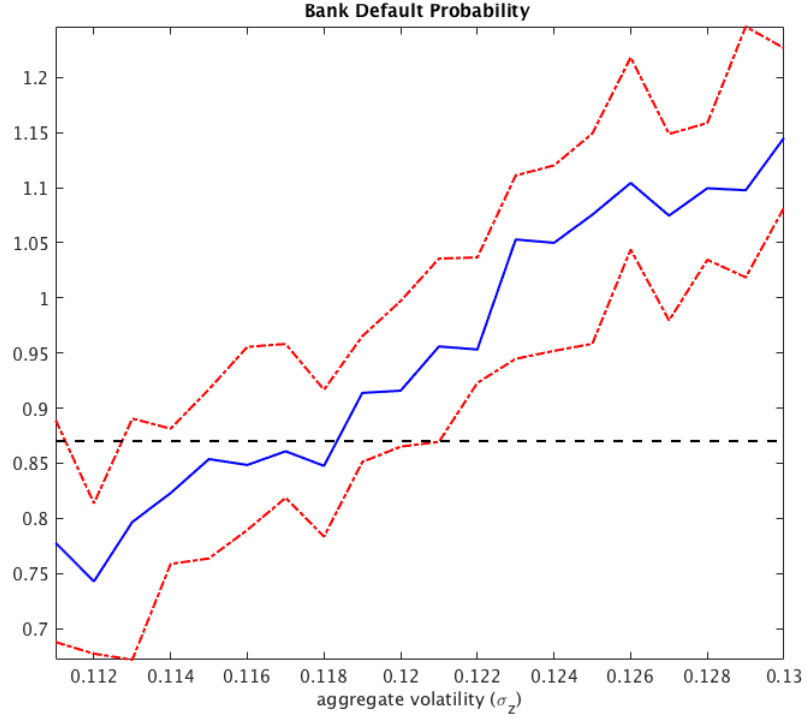
I examine the identification of parameters by conducting an exercise similar to Daruich [2018]. I use simulated method of moments to assess the relevance of our chosen target moments for the identification of parameters. The model has seven parameters that are internally calibrated to match seven data moments. The identification of some parameters relies on some key moments in the data. The identification exercise is conducted as follows. First, I draw a vector of candidate parameter values uniformly from the neighborhood of the calibrated value for $N_1 = 100,000$ times, and compute the implied moments in the model. Second, we associate each parameter with a target moment as listed in Table 1.2. Now there are N_1 simulated values for each parameter, as well as for the associated moment. Third, the vector of simulated values for each parameter is divided into $N_2 = 50$ quantiles. For each quantile, there are N_1/N_2 associated moments. I then compute the 25th, 50th, and 75th percentiles of the associated moment.

Figure (1.4) shows the identification of the key parameter σ_z , the standard deviation of the aggregate shock. The figure conveys several pieces of information about the identification. First, we claim that a moment is important for a param-

eter’s identification if the “confidence band” (i.e., the difference between the 25th and 75th percentiles) of the associated moment crosses the horizontal dotted line, which is the value of the moment in the data. Second, we interpret the slope of the confidence band as the sensitivity of the parameter to the associated moment. In other words, a steeper curve implies that the moment is more informative. Third, the width of the confidence band informs us about the relative importance of the remaining parameters. A wide band suggests that other parameters are important in affecting this moment. Figure (1.4) suggests that the moment of banks’ default probability is informative in identifying the key parameter σ_z . Intuitively, a higher variance of aggregate productivity makes it more likely that a bank will experience a wave of correlated firm defaults, which in turn causes it to default on its debt to the mutual fund.

1.6 Numerical Results

In this section, I numerically simulate the model economy to analyze its dynamic properties. The model is solved using local perturbation methods. I consider a banking crisis experiment in which banks suffer a significant loss in their capital. I first study the response of the economy to the shock in my full model. In particular, I highlight banks’ endogenous decisions in lending standards to illustrate the mechanism. Then I show how the response of macroeconomic variables to the shock is amplified and propagated through the endogenous lending standards channel. To do so, I compare the impulse responses to the financial shock under the full model



Notes: The figure shows the identification of parameter aggregate volatility σ_z using the simulated method of moments. The black horizontal line is the value of banks' probability of default in the data. The blue solid line is the 50th percentile of the simulated moments for any given value of σ_z . Similarly, the red dotted lines are the 25th and the 75th percentile of the simulated moments.

Figure 1.4: Simulated Method of Moments

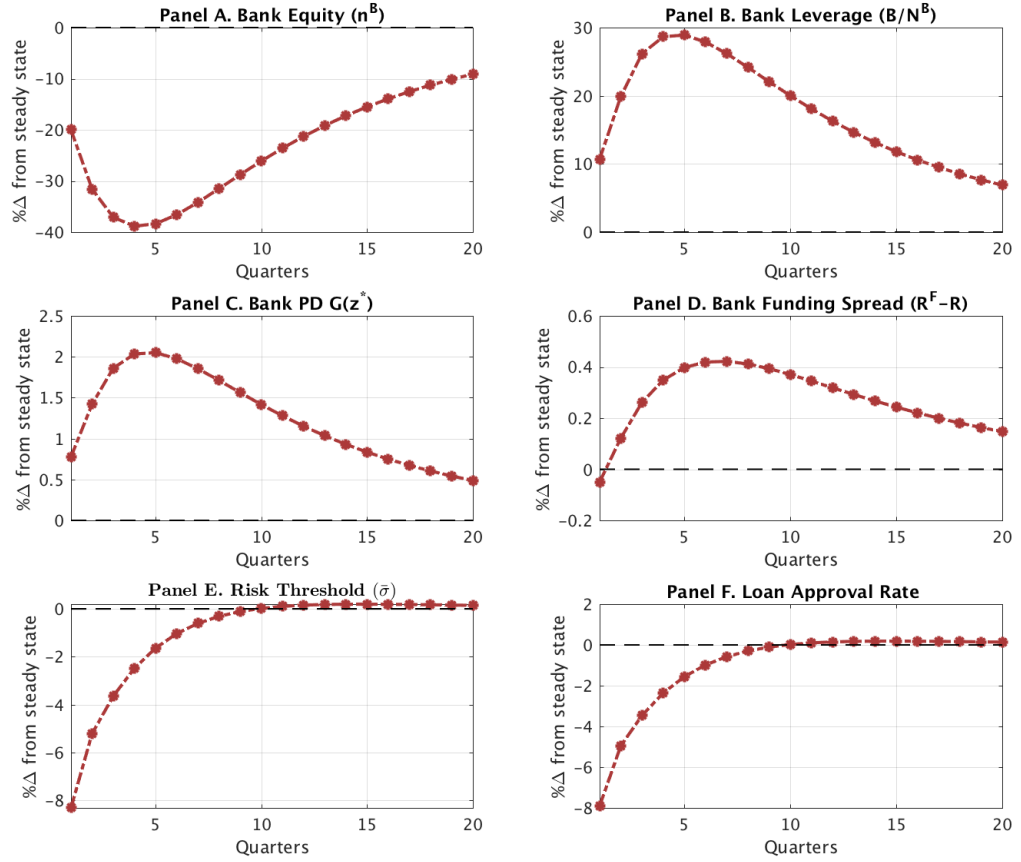
with those from a benchmark model, in which banks do not adjust lending standards and lend to all borrowers with a common loan contract.

1.6.1 Dynamic Effects of Financial Shocks

I simulate the response of the model economy to an exogenous shock that leads to a 20% loss of equity in the banking sector.²⁸ The shock can be interpreted as unexpected losses in banks' assets, such as losses in the sub-prime mortgage market, which are not modeled explicitly in this paper.

Figure (1.5) presents the impulse response functions of bank variables to the financial shock. Banks' equity falls immediately on impact. Banks' leverage ratio, $\frac{D+N^B}{N^B}$, rises and evolves as the mirror image of bank equity, as shown in Panel B. The response of the bank leverage ratio closely follows bank equity N^B because bank debt D , which equals the total deposits from households in equilibrium, mainly responds to the risk-free interest rate in general equilibrium and hence cannot be quickly adjusted. There will be more banks that are not able to repay their debt, which pushes up banks' default threshold z^* and probability of default ($G(z^*)$), as shown in Panel C. As a result, banks' funding cost $R^F - R$ goes up (Panel D), as banks' debtors require a higher spread to cover the higher bank default risk. This rise in the banks' funding cost triggers our main mechanism – banks tighten their lending standards and reject more borrowers (Panels E and F). It is worth noting that the response of bank equity is hump-shaped. Bank equity declines further in

²⁸Since banks in this model are identical ex-ante, it is equivalent to consider a shock to a representative bank.



Notes: The figure shows impulse response functions of aggregate variables to the financial shock. The size of the shock is calibrated such that banks lose 20% of equity.

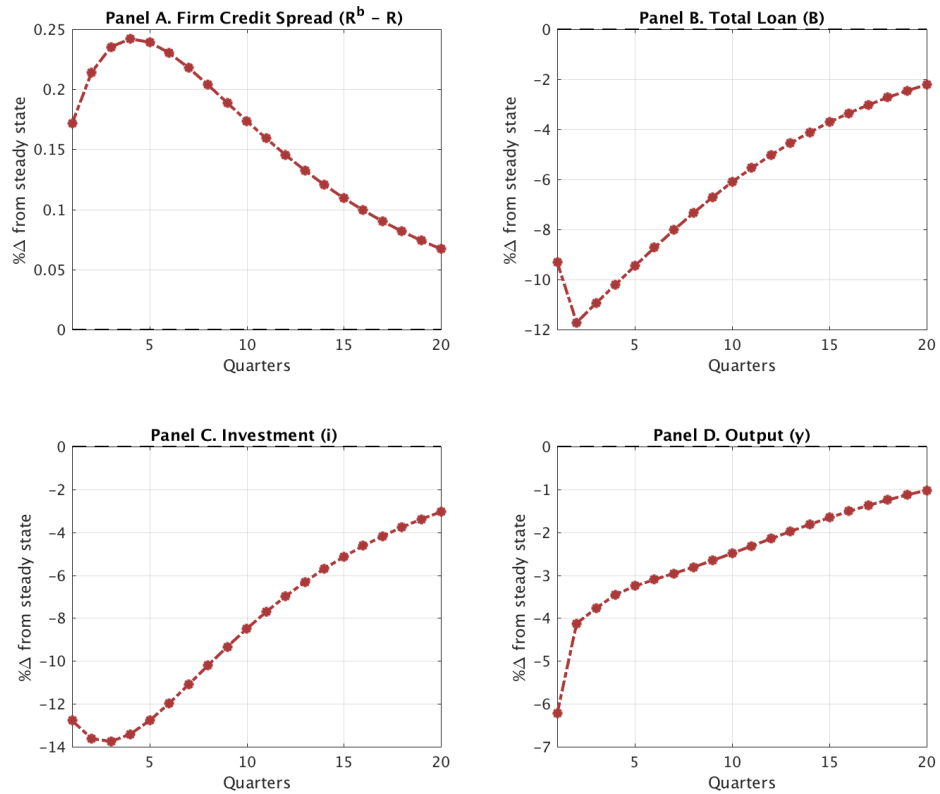
Figure 1.5: IRFs to Financial Shock - Bank Variables

response to the initial impact. This is due to the fact that bank equity is accumulated through retained earnings, and when adverse shocks lower project return, it also substantially reduces banks' profitability.

Figure (1.6) presents the impulse response functions of aggregate variables to the financial shock. My model captures two important channels that lead to the decline in aggregate credit supply. One is the rejection channel aforementioned. More borrowers are credit rationed and those rationed borrowers have to downsize their projects without access to bank financing. The other channel is the conventional price channel – firms' borrowing credit spread increases as shown in Panel A. Therefore, total borrowing declines (Panel B), which lowers investment (Panel C) and output (Panel D).

1.6.2 Comparison with the Benchmark Model

In this section, I examine the amplification and propagation mechanism by comparing the impulse response functions of the full model to those of a benchmark model in which banks do not reject borrowers. The benchmark model is a BGG model with an explicit banking sector, which is nested in the full model, with the only difference being no rejection choice in banks' problem. Such difference comes from the assumption in the benchmark model that banks cannot observe any information on borrowers' risk when deciding on loan contracts, in contrast to the assumption in the full model that banks have a binary signal on borrowers' risk. Therefore, in the benchmark model banks cannot differentiate borrowers in the loan contract.



Notes: The figure shows impulse response functions of aggregate variables to the financial shock. The size of the shock is calibrated such that banks lose 20% of equity.

Figure 1.6: IRFs to Financial Shock - Aggregate Variables

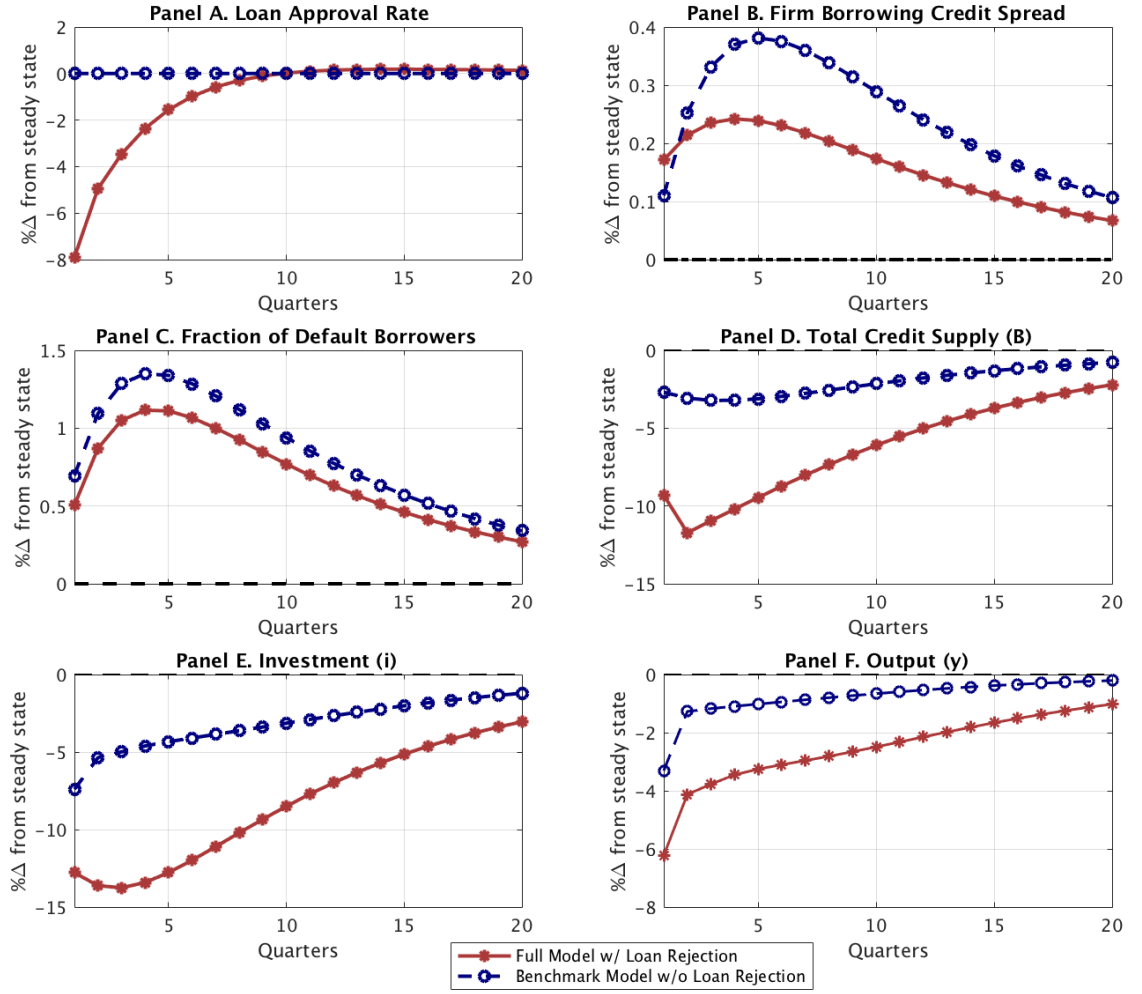
There will be no rejection and all entrepreneurs will take the same loan contract. I recalibrate the benchmark model to ensure that bank and firm leverages are the same in both models. The idea is to have both the corporate sector and the financial sector in the same state (characterized by the leverage) whether the model has or not lending standards. To do so, I alter values for L^E and L^B in the benchmark model. Appendix [A.2](#) provides a detailed description of the benchmark model.

Figure (1.7) presents the impulse response functions of several key variables to the financial shock under both models. Panels A and B of Figure (1.7) demonstrate two distinct channels in banks' response to capital shortfalls. In the benchmark model, banks do not reject borrowers; therefore the loan approval rate is 100% and does not respond to the financial shock. Instead, banks respond by charging higher lending rates, which raises firms' borrowing credit spread. Panel B shows that firm credit spreads increase by 40 basis points at their peak. The higher borrowing cost in turn lowers firms' credit demand and decreases total lending in equilibrium (Panel D). As a result, investment and output fall as shown in Panels E and F. This is the conventional price channel through which adverse financial shocks disrupt the real economy.

In contrast, there is a distinct credit rejection channel in the full model. The adverse shock to bank equity increases the share of loan applicants who are denied credit by 8 percentage points relative to the steady state. Those rejected borrowers are credit constrained, so they have to rely solely on their internal funds to invest. This contributes to the decline in total loan supply (Panel D), investment (Panel E), and output (Panel F). More importantly, loan rejection brings a compositional

effect in the banks' loan portfolio. The rejected borrowers are those with high risk. When banks reject more applicants, the pool of approved borrowers becomes less risky. This can be seen from Panel C, which shows that the fraction of defaulting borrowers responds less in the full model compared to the benchmark model. Therefore, banks can offer relatively lower loan interest rates to these safer borrowers, putting downward pressure on the lending rate. Meanwhile, the pass through of banks' rising funding costs still generates upward pressure on loan interest rates. The combined effect, as shown in Panel B, is that the lending rate still increases, but not as much as it does in the benchmark model. Panel B shows that the lending rate increases by about 25 basis points at its peak, less than its peak response in the benchmark model. This feature of my model echoes the motivation of this paper. During the last financial crisis, interest rate spreads on bank loans only increased mildly, while lending standards and loan rejection rates went up significantly. The results of my model are consistent with these findings.

The full model with the rejection channel yields larger amplification and propagation effects compared to the benchmark model. Panel E shows that the peak investment decline in the full model is about double of that in the benchmark model. Similarly, output in the full model decreases much more on impact, and the recovery is significantly slower.



Notes: The figure shows impulse response functions of bank and aggregate variables to the financial shock. The size of the shock is calibrated such that banks lose 20% of equity. Solid lines are from the full model with loan rejection. Dashed lines are from the benchmark model without loan rejection.

Figure 1.7: IRFs to Financial Shock - Comparison

1.7 Conclusion

The effect of changes in bank capital on the provision of bank credit is a key determinant of the linkage between the financial sector and the real economy. Understanding the underlying mechanisms has therefore been one of the most important research questions since the last financial crisis, and is also crucial for the design of monetary and macro-prudential policies in central banks and other policy institutions. This paper contributes to this line of research by concentrating on a nonprice channel, namely the lending standards. In contrast to existing models which mainly rely on price channels, this paper innovates by allowing banks to adjust lending standards. In the model, banks respond to adverse shocks not only by charging higher interest rates, but also by adjusting lending standards and denying credit applications.

In this paper, I propose a micro-founded theory in explaining banks' decisions on credit rejection. I embed this theory into a rich yet tractable quantitative DSGE model. The calibrated model matches the average approval rate of business loans, non-financial firms' and banks' probability of default and the loss given default of bank loans. I conduct a counter-factual exercise where banks suffer 20% equity losses, and compare dynamic responses in the full model to an otherwise identical benchmark model without credit rejection. I find that, when banks can adjust both the rejection margin and the interest rate margin as in the full model, responses of lending spreads on interest rates are milder. As a substitute, the rejection rate on bank loans increases significantly. This is consistent with the fact that interest rate

spreads were relatively flat during the recent crisis. Overall, the simulation results show that the credit rejection channel is quantitatively and qualitatively important for amplifying financial shocks.

The mechanism in this paper can be extended to study the disproportional impact on small and young businesses during the financial crisis and the contribution of this impact to the slow recovery in the aftermath. One prediction from the model is that the riskier firms are more likely to be credit rationed during the downturn. This prediction can have a distributional effect on heterogeneous firms. In particular, one can introduce a positive correlation between mean and variance in the distribution of firms' idiosyncratic productivity, to capture the idea that riskier businesses tend to have better growth prospects on average. With this new feature, the extended model would generate a misallocation of credit. Note that there is no social cost to lending to all borrowers, even risky ones, from the social planner's standpoint. Such an extension would also enhance our results and is expected to generate larger amplification.

Another possible avenue for future research is to examine how the mechanism in this paper might alter the transmission of (unconventional) monetary policies, relative to a model with only a price channel. For instance, during the last financial crisis, the Federal Reserve, as well as central banks in many other countries, implemented a number of programs to enhance the provision of funding to banks and to taper banks' rising cost of funds. One way to think about such policies in this model is to introduce monetary policies that affect the interest rate that the mutual fund pays on deposits. In a model with only a price channel, the effectiveness of

such policies can be evaluated by only looking at firms' borrowing spreads, which is a sufficient statistic for accessing the distress in the credit market. In contrast, the effect of such monetary policy in this paper's model would be transmitted through an additional channel, namely the credit rejection channel. For the same level of declining in lending spreads, financial conditions are more relaxed if loan approval rates increase.

Chapter 2: Capital Gaps, Risk Premia Dynamics and the Macroeconomy (coauthored with Fabian Lipinsky and Malgorzata Skibinska)¹

2.1 Introduction

The financial crisis and ongoing financial policies demonstrate the challenges in identifying and managing booms and busts in financial cycles. Alternating periods of excessive credit growth and credit crunch caused by fluctuations in bank capital have generated much concern on financial stability. Capital regulation as a result has also moved from constant risk-based capital requirements, as in the 2004 Basel II Accord, to a dynamic capital buffer framework as in the 2010 Basel III Accord.² In this paper we develop a rich framework to explore the cyclical nature of bank capital. We take a normative angle of bank capital analysis by asking what is the "natural" level of capital (or leverage) that banks would choose optimally in the absence of frictions. Through counterfactual exercises, we analyze how historical levels of bank capital implied by the data deviate from model-computed natural

¹Fabian Lipinsky: International Monetary Fund. Malgorzata Skibinska: Warsaw School of Economics

²See Nguyen (2014), Begenau (2018), Van den Heuvel (2016), and Begenau and Landvoigt (2017)

levels, and further decompose bank capital gaps into the contributions of different types of shocks. We lastly highlight the channels through which bank capital gaps affect the real economy.

We nest two parallel economies under one unified framework. The only difference between the two economies lies in the capital structure choices.³ In the calibrated economy, equity is not optimally chosen. Instead, we assume that equity dynamics follows a law of motion a la [Christiano et al. \[2014\]](#). We do not explicit model which types of frictions that prevent equity from the optimal level. Equity dynamics is calibrated to match the data. The other economy, which is termed as the "optimal" economy, features with optimal capital structure choices made by both firms and banks. Equity in the optimal economy is endogenously determined every period. In both economies, we focus on the interconnectedness between borrowers' and lenders' balance sheet dynamics. Deteriorating borrower balance sheets increases loan and security portfolio losses of lenders, weakening lenders' balance sheet conditions. At the same time, shortfall in financial intermediaries' capital leads to an increase in funding spreads of financial institutions, which is passed on to non-financial firms reflected as a rise in credit spreads and a reduction of credit. Hence, the various feedback effects between borrowers and lenders are not separable. This paper provides a framework to study the joint dynamics of their balance sheets, default frequencies, and associated risk premia.

We introduce a theory of optimal capital structure into a standard real business cycle framework. In this optimal economy, lenders (financial intermediaries) and

³We use equity, capital, and net worth interchangeably throughout this paper.

borrowers (non-financial firms) finance their expenditures by issuing equity and debt. The extent of equity financing versus debt financing depends on the trade-offs of each financing option. We depart from the Modigliani-Miller world, in which capital structure is indeterminate. Instead, following the corporate finance literature, we assume that debt has a tax advantage over equity, but there is a cost associated with debt default.⁴ The tradeoff between expected default costs and the tax advantage of debt generates an interior choice for the capital structure.

We use Bayesian methods as in An and Schorfheide (2007) to estimate the model on a sample of US macroeconomic time series from 1991Q1 to 2016Q4. The RBC model defined in this paper without financial frictions would have four shock processes, and could be estimated with standard aggregate data series such as GDP, consumption, investment and employment. In addition, we include two more shock processes, representing idiosyncratic risk and aggregate risk, along with two additional financial data series in our estimation, firms' credit spreads and financial funding spreads, to estimate how financial frictions in the model affect the macroeconomy.

Our econometric analysis finds that changes in both idiosyncratic risk and aggregate risk are important drivers of business cycle fluctuations. Upon impact, a surge in either risk shock creates a joint decline of investment, output, consumption and employment, but through different channels. The main operating channel for aggregate risk is through banks' funding cost. An increase in aggregate risk raises

⁴In reality, interests payments on debt are deductible from taxable corporate income, while firms' earnings are taxable.

the likelihood for banks to default. Banks' debt holders consequently require larger spreads on banks' debt, which in turn passes on to banks' lending spreads, reducing the credit extended to the economy. On the other hand, the main operating channel for idiosyncratic risk is through firms' credit spreads. An increase in idiosyncratic risk primarily leads to more firm (borrower) default, and hence generates higher costs for firm borrowing.

Our counterfactual analysis shows that responses of macroeconomic and financial variables to a given shock is generally smaller in the optimal economy than in the calibrated economy. Differences in the responses are larger in the case of risk shocks than in the case of a standard productivity shock. Following an increase in idiosyncratic (aggregate) risk, the fraction of defaulting firms (banks) rises, and firm (bank) equity declines. In response, however, firms (banks) will issue more equity to have a faster recovery in the optimal economy, whereas in the calibrated economy equity is accumulated only through constant retained earnings and recovers slowly. The differing speed in equity recovery has an impact on the dynamics of aggregate variables. The impulse responses show that aggregate series of output, investment and consumption in the optimal economy experience a milder magnitude of decline than in the calibrated economy. For the productivity shock, differences in the impulse responses between the two economies are almost negligible. This is because the productivity shock does not have much influence on the balance sheets of firms or banks.

We compute the historical evolution of bank capital gaps and conduct a variance decomposition analysis. We find that the bank capital gap exhibits counter-

cyclicality in general. There was a spike in the gap during the 2008-2009 financial crisis. The countercyclicality implied that banks' actual level of capital was much lower than their "desired" level during the crisis period. We also find that aggregate risk plays a significant role in driving the fluctuations of the bank capital gap.

2.2 Literature Review

This paper is related to the literature on bank capital requirements (Van den Heuvel (2008), De Nicolo et al. (2014), Nguyen (2014), Begenau (2018), Van den Heuvel (2016), Begenau and Landvoigt (2017)). The literature analyzes the welfare impact of bank capital through a regulatory angle. This paper abstracts from market frictions such as moral hazard that motivate the government intervention in the literature. The only friction in the model is the monitoring cost that drives the capital structure.

More broadly, this paper fits a strand of macroeconomic literature on the role of financial intermediation in the development of economic crises, including the seminal works of Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Bernanke, Gertler and Gilchrist (1999), and Gertler and Kiyotaki (2010), Gertler and Karadi (2011), He and Krishnamurthy (2012), Di Tella (2013), and Brunnermeier and Saniklov (2014).

This paper explores the role of uncertainty shocks in the dynamics of firms' and banks' balance sheets. Along the same line, [Christiano et al. \[2014\]](#) focus on the impact of idiosyncratic risk shocks on firms' funding costs. They find that the risk

shock can account for approximately 60 percent of fluctuations in aggregate output.

Structure of the paper The paper is structured as follows. Section 2.3 describes the model. Section 2.4 describes the estimation strategy and the data. We present the quantitative results in section 2.5, and conclude in section 2.6.

2.3 The Model

Our model consists of a representative household, firms, financial intermediaries (banks), and a mutual fund. Similar to [Gertler and Kiyotaki \[2010\]](#), firms are located on islands. There is a representative bank in each island. Banks only lends to firms in the same island, and borrow from the mutual fund. The mutual fund takes deposit from the household and lend to banks.

2.3.1 Households

There is a unit measure of identical households in this economy. The representative household chooses consumption C_t , labor supply L_t^H and deposit saving D_{t+1}^H to maximize the expected lifetime utility given by

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s z_{c,t} \left(\ln(C_{t+s} - hC_{t+s-1}) - z_{n,t} \frac{\psi_L}{1+\varphi} (L_{t+s}^H)^{1+\varphi} \right)$$

subject to the budget constraint

$$C_t + D_{t+1}^H / R_t = W_t L_t^H + D_t^H + \Pi_t \tag{2.1}$$

where $U(\cdot)$ is the period felicity utility, R_t the risk-free interest rate, W_t the wage rate, and Π_t the total profits and dividends transferred from the entrepreneurs and banks. $z_{c,t}$ and $z_{n,t}$ are the preference and labor dis-utility shock, respectively. Parameters β, h, ψ_L , and φ are the discount factor, a consumption habit parameter, a scaling parameter on the marginal utility between consumption and labor, and the inverse of the Frisch elasticity of labor supply, respectively.

The first order conditions with respect to consumption, labor, and savings yield the following optimality conditions:

$$\mathbb{E}_t(M_{t+1}R_t) = 1 \quad (2.2)$$

$$U_C(C_t, L_t^H)W_t = U_L(C_t, L_t^H) \quad (2.3)$$

$$M_{t+1} \equiv \beta U_C(C_{t+1}, L_{t+1}^H)/U_C(C_t, L_t^H) \quad (2.4)$$

where M_{t+1} denotes the stochastic discount factor of households.

2.3.2 Firms

There is a continuum of perfectly competitive firms on each island, which are identical ex ante and differ ex post only in their realization of idiosyncratic and aggregate shocks. At the end of period t , new firms are born and purchase capital K_{t+1} for use in period $t + 1$. Firms finance capital expenditures by issuing equity N_t^F and debt B_{t+1} . In period $t + 1$, after the realization of shocks, firms decide on labor demand and production, and then sell back their capital in the competitive capital market. At this point there are two possible scenarios: the firm value V_{t+1}^F

is larger than outstanding debt, in which case the firm is able to repay the debt in full and all the remaining value goes to the equityholders as dividends; or the firm value is less than outstanding debt. In this case debt holders have higher priority than equityholders: debt holders will capture the firm value net of any bankruptcy cost, and equityholders are wiped out.

2.3.2.1 Firm Production

Firms are perfectly competitive. They are the consumption goods producers in this economy. All firms operate the same constant returns to scale Cobb-Douglas production function using effective capital and labor:

$$Y_t = \tilde{K}_t^\alpha (A_t L_t)^{1-\alpha} \quad (2.5)$$

where A_t is the labor-augmented total factor productivity (TFP), and Y_t , \tilde{K}_t , and L_t are the individual firm's output, effective capital, and labor demand, respectively. Since each firm is identical ex-ante and makes the same decision choices, we suppress firm subscript j for simplicity so that the firm becomes representative.

At time $t - 1$, the entrepreneur purchases raw capital, denoted by K_t , for use at t . The actual quantity of capital that can be used for production at time t , denoted by the effective capital \tilde{K}_t , is random. In particular, for K_t units of capital the entrepreneur purchases at time $t - 1$, the total amount of effective capital ready for production at time t is $\tilde{K}_t = x_t K_t$, where x_t is a composition of two shocks to the firm's capital stock. Specifically, x_t comprises a firm-specific shock component

ε_t and an island-wide shock component z_t , and $x_t = \varepsilon_t z_t$. The firm has $(1 - \delta)\tilde{K}_t$ units of undepreciated capital available for resale after production.

2.3.2.2 Firm Value

At time t , the firm chooses labor after the realization of the idiosyncratic shock and the island-wide shock, given the capital stock purchased from last period. Hence, we can solve the firm's labor choice through a static profit maximization problem. Given the capital stock, the aggregate wage and the realized shocks, the firm chooses labor L_t to maximize its profit Π_t :

$$\max_{L_t} \Pi_t = \tilde{K}_t^\alpha (A_t L_t)^{1-\alpha} - W_t L_t \quad (2.6)$$

Profit maximization yields the optimal labor demand as a function of capital, given by

$$L_t = \tilde{K}_t \left(\frac{A_t^{1-\alpha} (1-\alpha)}{W_t} \right)^{1/\alpha} \quad (2.7)$$

Let V_t^E denote the firm's total value at period t after production (but before making loan repayments), where the superscript E denotes entrepreneur. The entrepreneur who experienced shock x_t is left with $(1 - \delta)x_t K_t$ units of capital after depreciation. This capital is sold in competitive markets at the price Q_t . Hence the firm's value is the sum of maximized profit plus the value of undepreciated effective capital: $V_t^E = \Pi_t + (1 - \delta)Q_t \tilde{K}_t$. Substituting for the optimal labor demand and rearranging

gives the firm's value as

$$V_t^E = \left[\alpha \left(\frac{A_t(1-\alpha)}{W_t} \right)^{(1-\alpha)/\alpha} + (1-\delta)Q_t \right] x_t K_t \quad (2.8)$$

Let R_t^k denote the return on effective capital, which is common across all entrepreneurs:

$$R_t^k \equiv \frac{\alpha \left(\frac{A_t(1-\alpha)}{W_t} \right)^{(1-\alpha)/\alpha} + (1-\delta)Q_t}{Q_{t-1}} \quad (2.9)$$

Therefore, we write the firm's value in a compact expression as

$$V_t^E = x_t R_t^k Q_{t-1} K_t = \varepsilon_t z_t R_t^k Q_{t-1} K_t \quad (2.10)$$

That is, the firm's individual return on raw capital is the product of the idiosyncratic shock, the island-specific shock and the common return on effective capital: $\varepsilon_t z_t R_t^k$.

2.3.2.3 Firms default

Let $Q_t K_{t+1}$ be the firm's capital expenditure. The firm finances the expenditure partly through equity and partly through debt. We will discuss in detail in the following section on the firm's financing choice. The firm's balance sheet identity is given by

$$B_t + N_t^E = Q_t K_{t+1} \quad (2.11)$$

Assets	Liabilities
$Q_t K_{t+1}$	$\frac{B_t}{N_t^E}$ Equity

Figure 2.1: Firms' Balance Sheet

The amount B_t is borrowed from the bank in the form of debt. The firm promises to repay the bank at the face value $R_{t+1}^b B_t$ at time $t+1$, where R_{t+1}^b is the gross interest rate. After the realization of idiosyncratic and aggregate shocks at time $t+1$, if the total value of the firm falls below the promised amount of debt repayment, the firm defaults. Conditions for firm default are characterized as follows:

$$\underbrace{\varepsilon_{t+1} z_{t+1} R_{t+1}^k Q_t K_{t+1}}_{\text{Firm Value}} \leq \underbrace{R_{t+1}^b B_t}_{\text{Loan repayment}} \quad (2.12)$$

I use \bar{x}_{t+1} to denote the entrepreneur's default threshold on total productivity, which is a product of the idiosyncratic and aggregate shock, given by

$$\varepsilon_{t+1} z_{t+1} \leq \bar{x}_{t+1} \equiv \frac{R_{t+1}^b B_t}{R_{t+1}^k Q_t K_{t+1}} \quad (2.13)$$

For realizations of x_{t+1} above \bar{x}_{t+1} the borrower pays $R_{t+1}^b B_t$ to the bank. For realizations below \bar{x}_{t+1} , the bank seizes the borrower's assets after paying a proportional monitoring cost μ_E . Firms' default can be attributed to an adverse idiosyncratic shock ε_{t+1} to the firm, or an adverse aggregate shock z_{t+1} affecting all firms, or a combination of both. The two shocks have the same impact on an individual firm's

ability to repay its debt, but they have different implications for the bank, which we will illustrate in the next section. Conditional on the aggregate shock z_{t+1} , the defaulting firms are those with realized idiosyncratic shocks lower than \bar{x}_{t+1}/z_{t+1} . In other words, \bar{x}_{t+1}/z_{t+1} is the default threshold for the idiosyncratic shock.

2.3.3 Financial Intermediaries

Financial intermediaries provide loans B_t to non-financial firms at gross lending rate R_t^b . Given the loan contract, the bank's revenue from this contract is either $R_t^b B_t$ when firms are able to make full repayment or the value of the firm when the firm defaults. For a given aggregate shock z_{t+1} , the bank's revenue is

$$\begin{aligned} & \int_{\frac{\bar{x}_{t+1}}{z}}^{\infty} R_t^b B_t dF(\varepsilon) + (1 - \mu_E) \int_0^{\frac{\bar{x}_{t+1}}{z}} \varepsilon z R_{t+1}^k Q_t K_{t+1} dF(\varepsilon) \\ &= \left[\int_{\frac{\bar{x}_{t+1}}{z}}^{\infty} \bar{x}_{t+1} dF(\varepsilon) + (1 - \mu_E) \int_0^{\frac{\bar{x}_{t+1}}{z}} \varepsilon z dF(\varepsilon) \right] R_{t+1}^k Q_t K_{t+1} \\ &\equiv \mathcal{S}^B(\bar{x}_{t+1}, z_{t+1}) R_{t+1}^k Q_t K_{t+1} \end{aligned}$$

The balance sheet of banks is shown below. The bank finances its asset B_t with debt D_t and equity N_t^B . We will discuss the financing options in detail in the following section.

$$B_t = N_t^B + D_t \tag{2.14}$$

For a given loan contract, the bank's revenue positively depends on the realized value of aggregate shock z_{t+1} . When the realized value of z_{t+1} is too low, the bank may not be able to repay its debt fully, in which case the bank defaults. The

Assets	Liabilities
B_t	$\frac{D_t}{N_t^B}$ Equity

Figure 2.2: Banks' Balance Sheet

threshold value of the aggregate productivity z_{t+1}^* for bank default is characterized by the following equation:

$$\mathcal{S}^B(\bar{x}_{t+1}, z_{t+1}^*) R_{t+1}^k Q_t K_{t+1} = R_{t+1}^d D_t \quad (2.15)$$

where $\mathcal{S}^B(\bar{x}_{t+1}, z_{t+1}^*)$ is banks' *realized* share of the total return, conditional on the realized value of z_{t+1}^* , and R_{t+1}^d is the interest rate on bank's debt, which will be determined in the mutual fund's problem. This condition states that the realized value of bank revenue (the left hand side of 2.15) is just paying off the debt borrowed from the mutual fund (the right hand side of 2.15), leaving the whole equity being wiped out.

2.3.4 The Mutual Fund

The mutual fund takes deposit from the household and lends to a continuum of banks. The participation constraint is given by

$$\mathbb{E}_t \{ M_{t+1} [(1 - G(z_{t+1}^*)) R_{t+1}^d D_t + (1 - \mu_B) \underline{\mathcal{S}}^B(\bar{x}_{t+1}, z_{t+1}^*) R_{t+1}^k Q_t K_{t+1}] \} = D_t \quad (2.16)$$

where $0 < \mu_B < 1$ is the monitoring cost when banks default. The first term of the above expression is banks' full payment when not in default, i.e., the face value of bank's debt $R_{t+1}^d D_t$. When banks default, however, banks' debt holders claim the banks' assets, net of the proportional monitoring cost μ_B , denoted by the second term. The left hand side is the present discounted value of total lending, and the right hand side is its total lending. The equality sign indicates the break-even condition for the mutual fund.

2.3.5 Joint Capital Structure Choice

In this section, we describe how capital structure is determined for both firms and banks. Firms and banks finance their expenditure using both equity and debt. We set up two cases in which the equity financing is determined in two different ways. In the optimal capital structure, the amount of equity to issue is an endogenous variable that agents (firms and banks) can choose each period. In the second case, equity is not a choice variable. Instead, it follows a law of motion and is calibrated to match the data.

2.3.5.1 Case I: Optimal Capital Structure

Firms finance expenditure partly with debt (bank loans) and partly with equity. Similar to [Gourio \[2013\]](#) and [Jermann and Quadrini \[2012\]](#), capital structure in this paper is driven by the tradeoff between debt financing and equity financing. Specifically, debt has a tax advantage over equity such that only the equity return

is taxable, but debt holders are subject to default costs. Debt holders recover a fraction $1 - \mu_E$ of the firm value upon default, where $0 < \mu_E < 1$ is the default cost such as monitoring or verification costs. On the other hand, equityholders' return will be taxed at the rate of $0 < \tau < 1$.

Equity issuance is assumed to be costless. When $\mu_E = \tau = 0$, firms' capital structure is indeterminate. When $\tau = 0$, the firm finances only through equity since debt has no tax advantage. When $\mu_E = 0$, the firm finances only through debt, since default is not costly.

The present value of firms' expected discounted net equity is

$$\mathbb{E}_t [M_{t+1} \mathcal{S}^E(\bar{x}_{t+1}) R_{t+1}^k Q_t K_{t+1}] (1 - \tau) - N_t^E \quad (2.17)$$

where $\mathcal{S}^E(\bar{x}_{t+1})$ denotes the share of return belonging to the firm and is given by

$$\mathcal{S}^E(\bar{x}_{t+1}) \equiv \int_0^\infty \left[\int_{\frac{\bar{x}_{t+1}}{z}}^\infty \varepsilon dF(\varepsilon) - (1 - F(\frac{\bar{x}_{t+1}}{z})) \frac{\bar{x}_{t+1}}{z} \right] z dG(z)$$

The optimal financial contract is to choose $(K_{t+1}, \bar{x}_{t+1}, z_{t+1}^*, R_{t+1}^d, B_t, D_t)$ to maximize firms' profit subject to bank's participation constraint

$$\mathbb{E}_t \left\{ M_{t+1} \left[\bar{\mathcal{S}}^B(\bar{x}_{t+1}, z_{t+1}^*) R_{t+1}^k Q_t K_{t+1} - (1 - G(z_{t+1}^*)) R_{t+1}^d D_t \right] \right\} (1 - \tau) - B_t + D_t = 0 \quad (2.18)$$

and the mutual fund's zero profit condition, given by

$$\mathbb{E}_t \left\{ M_{t+1} \left[(1 - G(z_{t+1}^*)) R_{t+1}^d D_t + (1 - \mu_B) \underline{\mathcal{S}}^B(\bar{x}_{t+1}, z_{t+1}^*) R_{t+1}^k Q_t K_{t+1} \right] \right\} = D_t \quad (2.19)$$

where z_{t+1}^* is characterized by the following equation:

$$\mathcal{S}^B(\bar{x}_{t+1}, z_{t+1}^*) R_{t+1}^k Q_t K_{t+1} = R_{t+1}^d D_t \quad (2.20)$$

2.3.5.2 Case II: Law of Motion for Equity

In this case, the financial contract is to choose $\{\bar{x}_{t+1}, K_{t+1}\}$ to maximize entrepreneurs' expected return, given by

$$\begin{aligned} & \mathbb{E}_t \int_0^\infty \left[\int_{\frac{\bar{x}_{t+1}}{z}}^\infty \varepsilon dF(\varepsilon) - (1 - F(\frac{\bar{x}_{t+1}}{z})) \frac{\bar{x}_{t+1}}{z} \right] z dG(z) R_{t+1}^k Q_t K_{t+1} \\ & \equiv \mathbb{E}_t [\mathcal{S}^E(\bar{x}_{t+1}) R_{t+1}^k Q_t K_{t+1}] \end{aligned} \quad (2.21)$$

where $\mathcal{S}^E(\bar{x}_{t+1}) \equiv \int_0^\infty \mathcal{S}^E(\bar{x}_{t+1}, z_{t+1}) dG(z_{t+1})$ is entrepreneur's expected share of return, and subject to the bank's participation constraint

$$\bar{\mathcal{S}}^B(\bar{x}_{t+1}, z_{t+1}^*) R_{t+1}^k Q_t K_{t+1} - (1 - G(z_{t+1}^*)) R_{t+1}^d D_t = R_t N_t^B \quad (2.22)$$

where $\bar{\mathcal{S}}^B(\bar{x}_{t+1}, z_{t+1}^*) \equiv \int_{z_{t+1}^*}^\infty \mathcal{S}^B(\bar{x}_{t+1}, z_{t+1}) dG(z_{t+1})$ is banks' expected share of return when they do not default. R_{t+1}^d is defined by the mutual fund's zero profit

condition, given by

$$(1 - G(z_{t+1}^*))R_{t+1}^d D_t + (1 - \mu_B)\underline{\mathcal{S}}^B(\bar{x}_{t+1}, z_{t+1}^*)R_{t+1}^k Q_t K_{t+1} = R_t D_t \quad (2.23)$$

where $\underline{\mathcal{S}}^B(\bar{x}_{t+1}, z_{t+1}^*) \equiv \int_0^{z_{t+1}^*} \mathcal{S}^B(\bar{x}_{t+1}, z) dG(z)$ is bank's expected share of return when it defaults. We can combine the above two constraints (with Lagrangian multiplier λ_{t+1}) as

$$\left[\bar{\mathcal{S}}^B(\bar{x}_{t+1}, z_{t+1}^*) + (1 - \mu_B)\underline{\mathcal{S}}^B(\bar{x}_{t+1}, z_{t+1}^*) \right] R_{t+1}^k Q_t K_{t+1} = R_t (Q_t K_{t+1} - N_t^E) \quad (2.24)$$

Equivalently, we can write the above equation as

$$\begin{aligned} N_t^E &= \left[1 - \left[\bar{\mathcal{S}}^B(\bar{x}_{t+1}, z_{t+1}^*) + (1 - \mu_B)\underline{\mathcal{S}}^B(\bar{x}_{t+1}, z_{t+1}^*) \right] \frac{R_{t+1}^k}{R_t} \right] Q_t K_{t+1} \\ &= \Phi(\bar{x}_{t+1}, z_{t+1}^*) Q_t K_{t+1} \end{aligned}$$

where $\frac{1}{\Phi(\bar{x}_{t+1}, z_{t+1}^*)}$ is entrepreneurs' asset-to-equity ratio, i.e., the leverage ratio. The zero profit conditions on other agents put a constraint on the maximum leverage of entrepreneurs. This characterization is isomorphic to the contracting problem in [Bernanke et al. \[1999\]](#). The difference is that there is an additional endogenous variable z_{t+1}^* in the leverage ratio. The contract will also affect banks' default probability through the choice of z_{t+1}^* , where z_{t+1}^* is characterized by the following equation:

$$\mathcal{S}^B(\bar{x}_{t+1}, z_{t+1}^*) R_{t+1}^k Q_t K_{t+1} = R_{t+1}^d (Q_t K_{t+1} - N_t^E - N_t^B) \quad (2.25)$$

where $\mathcal{S}^B(\bar{x}_{t+1}, z_{t+1}^*)$ is banks' *realized* share of return, conditional on the realized value $z_{t+1} = z_{t+1}^*$.

The optimal contract is to choose $\{\bar{x}_{t+1}, K_{t+1}\}$ to maximize equation (2.21) subject to equation (2.24), where z_{t+1}^* is defined by equation (2.25) and R_{t+1}^d is defined by equation (2.23). Let γ_t , λ_t and ν_t be the Langragian multipliers associated with (2.23), (2.24) and (2.25), respectively.

The evolution of equity in this economy follows a law of motion as in [Christiano et al. \[2014\]](#). Firms' net worth in period $t + 1$ is comprised of retained earnings and labor income. Entrepreneurs are assumed to supply L^E units of labor inelastically each period. This is a standard assumption in the literature to ensure equity always being positive. I assume that entrepreneurs have unit measure and that they pool resources at the end of each period. I also assume that entrepreneurs retain a fraction γ^E of earnings and rebate the remaining share $(1 - \gamma^E)$ as a dividend to households. Therefore, the net worth of an entrepreneur at the start of period $t + 1$ (or equivalently the end of period t) evolves as follows:

$$N_{t+1}^E = \gamma^E V_t^E + W_t L^E$$

Profits rebated to households as dividends are:

$$\Pi_t^E = (1 - \gamma^E) V_t^E$$

Similarly, banks' realized profit at the end of period t is:

$$V_t^B = \left[\int_{z_t^*}^{\infty} \mathcal{S}^B(\bar{x}_t, z) dG(z) \right] R_t^k Q_t K_t - (1 - G(z_t^*)) R_t^d D_{t-1}$$

At the end of each period, banks pay a fraction $(1 - \gamma^B)$ of their profits as dividends to the household. The remaining fraction γ^B is injected into new equity as retained earnings. The banker supplies L^B units of labor inelastically. Banks' equity at the start of period $t + 1$ is the sum of retained earnings and bankers' labor income:

$$N_{t+1}^B = \gamma^B V_t^B + W_t L^B; \quad \Pi_t^B = (1 - \gamma^B) V_t^B$$

2.3.6 New Capital Producers

The perfectly competitive capital producer transforms final consumption goods into capital. The production of new capital is subject to adjustment costs. In particular, capital producers take $(1 + S(\frac{I_t}{I_{t-1}}))I_t$ consumption goods and transform them into I_t investment goods that are sold at price Q_t . $S(\frac{I_t}{I_{t-1}})$ is the convex adjustment cost, which satisfies $S(1) = S'(1) = 0$ and $S''(1) \equiv \zeta > 0$. The capital producer's objective function is

$$\max_{I_t} \sum_{t=0}^{\infty} \beta^t M_{0,t} [Q_t I_t - (1 + S(\frac{I_t}{I_{t-1}})) I_t] \quad (2.26)$$

where $M_{0,t}$ is the stochastic discount factor. The first order condition of the capital producer's optimization problem is

$$Q_t = 1 + S\left(\frac{I_t}{I_{t-1}}\right) + \frac{I_t}{I_{t-1}} S'\left(\frac{I_t}{I_{t-1}}\right) - \beta E_t \left[M_{t,t+1} \left(\frac{I_{t+1}}{I_t}\right)^2 S'\left(\frac{I_{t+1}}{I_t}\right) \right] \quad (2.27)$$

where $S\left(\frac{I_t}{I_{t-1}}\right) = (z_{i,t} \frac{I_t}{I_{t-1}} - 1)^2$ and $z_{i,t}$ is an exogenous shock to capital adjustment cost.

2.3.7 Market Clearing

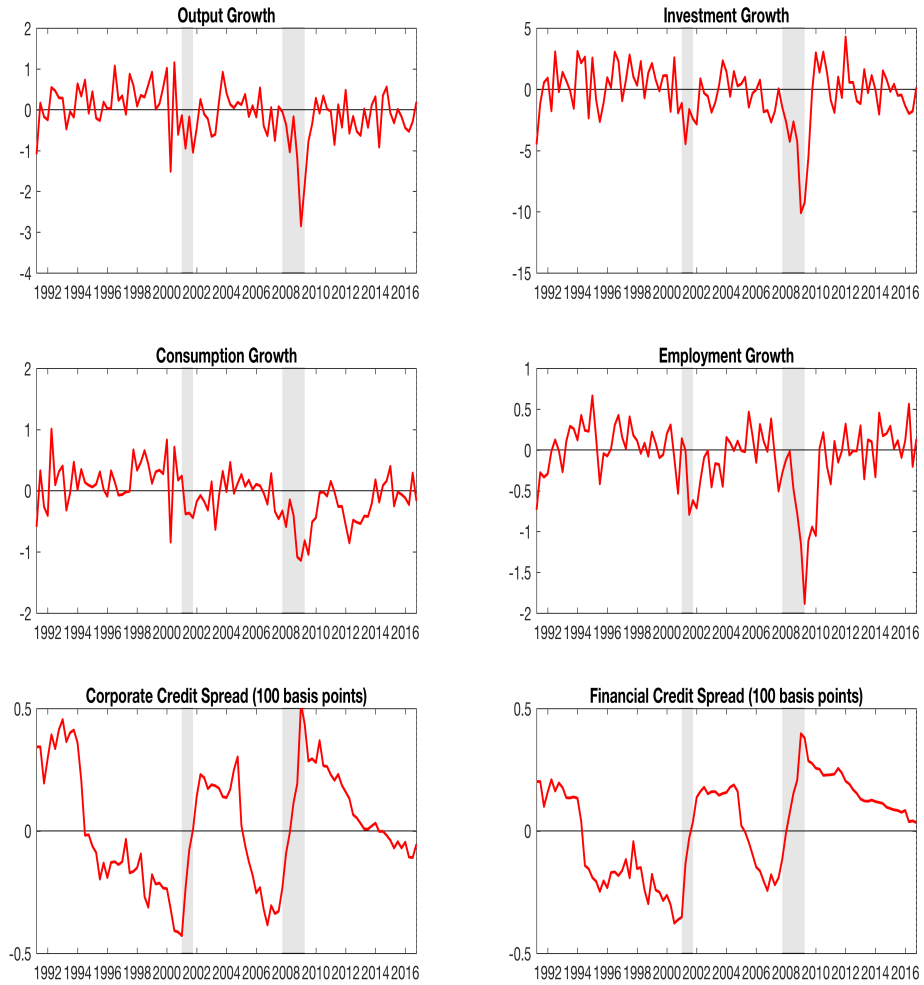
The funds market clears when household deposits equal the mutual fund's total lending, i.e., $D_t = D_t^H$. The economy-wide resource constraint holds when the total output equals to the sum of total consumption, investment plus adjustment costs.

2.4 Parameterization

In this section, we discuss how we set values for parameters in the model. We fit the second model economy, i.e. the model with the law of motion equity, with data on the United States economy on a quarterly basis. The premise We separate parameters into two groups. In the first group, parameters are set to follow conventional values in the literature. Parameters in the second group are estimated with standard Bayesian estimation techniques as in An and Schorfheide (2007). The estimation strategy works as follows. We estimate parameters governing the exogenous shocks. The model is matched to the data by maximizing the likelihood of observing the re-

alized data. We use four macroeconomic and two financial variables to identify these parameters. The macroeconomic series include output, consumption, employment and investment. We add two additional financial variables, firms' credit spreads ($R^b - R$) and banks' funding spreads ($R^d - R$), to match the financial side of the model economy.

We use US quarterly data between 1991Q1 to 2016Q3. GDP, consumption and investment data are from the Bureau of Economic Analysis. The data on employment is from the Bureau of Labor Statistics. Firms' credit spread is computed as the difference between the Baa corporate bond yield and the constant 1-year treasury bond yield. The financial credit spread is computed as the difference between the average US financial bond yield and the 3-month Treasury bill yield.



Notes: The figure shows the six data series used for estimation. All series are detrended. The four macroeconomic series are quarterly growth rates. Shaded areas denote NBER recessions.

Figure 2.3: Data Series for Estimation

2.4.1 Calibrated Parameters

Table 2.1 shows externally calibrated parameters. These parameters follow standard values in the literature. The tax rate $\tau = 0.2$ is similar to the tax rate on capital gains. The default cost parameter $\mu = 0.5$ reflects that the average recovery rate of bank loans is about 50%.

Parameter	Value	Description
β	0.996	Discount factor
α	0.35	Capital share in firm production
δ	0.025	Capital depreciation rate
φ	1	Inverse labor supply elasticity
ζ	0.5	Investment adjustment costs
h	0.7	Consumption habits
ψ_L	0.3	Disutility weight on labor
μ	0.5	Default cost
τ	0.2	Tax rate

Table 2.1: Calibrated Parameters

2.4.2 Estimated Parameters

We feed the model six data series and estimate six shock processes in total. All exogenous shocks follow an auto-regressive process of order one: productivity A_t , preferences $z_{c,t}$, labor disutility $z_{n,t}$, investment $z_{i,t}$, idiosyncratic risk $\sigma_{\varepsilon,t}$, and aggregate risk $\sigma_{z,t}$. We estimate the autocorrelation coefficients ρ and the standard deviations of each shock. Additionally, we estimate the steady state level of id-

idiosyncratic risk $\sigma_{\varepsilon,ss}$ and aggregate risk $\sigma_{z,ss}$. The innovations $\varepsilon_{j,t}$ follow a standard normal distribution.

$$\ln(A_t) = \rho_A \ln(A_{t-1}) + \sigma_A \varepsilon_{A,t} \quad (2.28)$$

$$\ln(z_{c,t}) = \rho_c \ln(z_{c,t-1}) + \sigma_c \varepsilon_{c,t} \quad (2.29)$$

$$\ln(z_{n,t}) = \rho_n \ln(z_{n,t-1}) + \sigma_n \varepsilon_{n,t} \quad (2.30)$$

$$\ln(z_{i,t}) = \rho_i \ln(z_{i,t-1}) + \sigma_i \varepsilon_{i,t} \quad (2.31)$$

$$\ln(\sigma_{\varepsilon,t}) = (1 - \rho_{\varepsilon}) \ln(\sigma_{\varepsilon,ss}) + \rho_{\varepsilon} \ln(\sigma_{\varepsilon,t-1}) + \sigma_{\varepsilon} \varepsilon_{\varepsilon,t} \quad (2.32)$$

$$\ln(\sigma_{z,t}) = (1 - \rho_z) \ln(\sigma_{z,ss}) + \rho_z \ln(\sigma_{z,t-1}) + \sigma_z \varepsilon_{z,t} \quad (2.33)$$

Table 2.2 summarizes the priors and also reports the posterior modes and standard deviations of the estimated parameters. The choice of the priors (mean and distribution) for the parameters of the model follows standard literature (Del Negro et al. [2011], Christiano et al. [2014]). Autocorrelation coefficients are assumed to follow beta distributions with Beta(0.9, 0.2) priors. The priors on the standard deviations of the innovations are inverse-gammas of type 2 with InvGamma(0.01, 0.002). We set the priors for the steady state levels of idiosyncratic risk and aggregate risk as $\mathbb{N}(0.2, 0.025)$ and $\mathbb{N}(0.1, 0.025)$, respectively.

Parameter	Description	Prior mean	Post mode	Post Std. Dev.	Dist.
Autocorr. Coefficients					
ρ_A	technology shock	0.9	0.8804	0.0293	beta
ρ_c	preference shock	0.9	0.9367	0.0469	beta
ρ_i	investment shock	0.9	0.7535	0.0522	beta
ρ_n	labor shock	0.9	0.9634	0.0170	beta
ρ_F	idiosyn. risk shock	0.9	0.9250	0.0216	beta
ρ_B	agg. risk shock	0.9	0.9700	0.0124	beta
Standard Deviations					
σ_A	technology shock	0.010	0.0077	0.0005	invga
σ_c	preference shock	0.010	0.0087	0.0016	invga
σ_i	investment shock	0.010	0.0313	0.0048	invga
σ_n	labor shock	0.010	0.0174	0.0011	invga
σ_ε	idiosyn. risk shock	0.010	0.0190	0.0025	invga
σ_z	agg. risk shock	0.010	0.0327	0.0040	invga
Steady State					
$\sigma_{\varepsilon,ss}$	idiosyn. risk shock	0.200	0.2759	0.0157	norm
$\sigma_{z,ss}$	agg. risk shock	0.100	0.1361	0.0091	norm

Table 2.2: Priors and Posterior Estimates for the Model Parameters

2.5 Quantitative Results

In this section we describe the impulse responses of macroeconomic and financial variables to various single-factor shocks. We show how these macroeconomic variables respond differently in the counterfactual economy where capital structures are optimally chosen (we call this the “optimal economy”), compared to the estimated model using real data (we call it the “estimated economy”). We then conduct historical decompositions of financial variables to compare the importance of various shocks in driving the fluctuations of these variables.

2.5.1 Impulse Responses

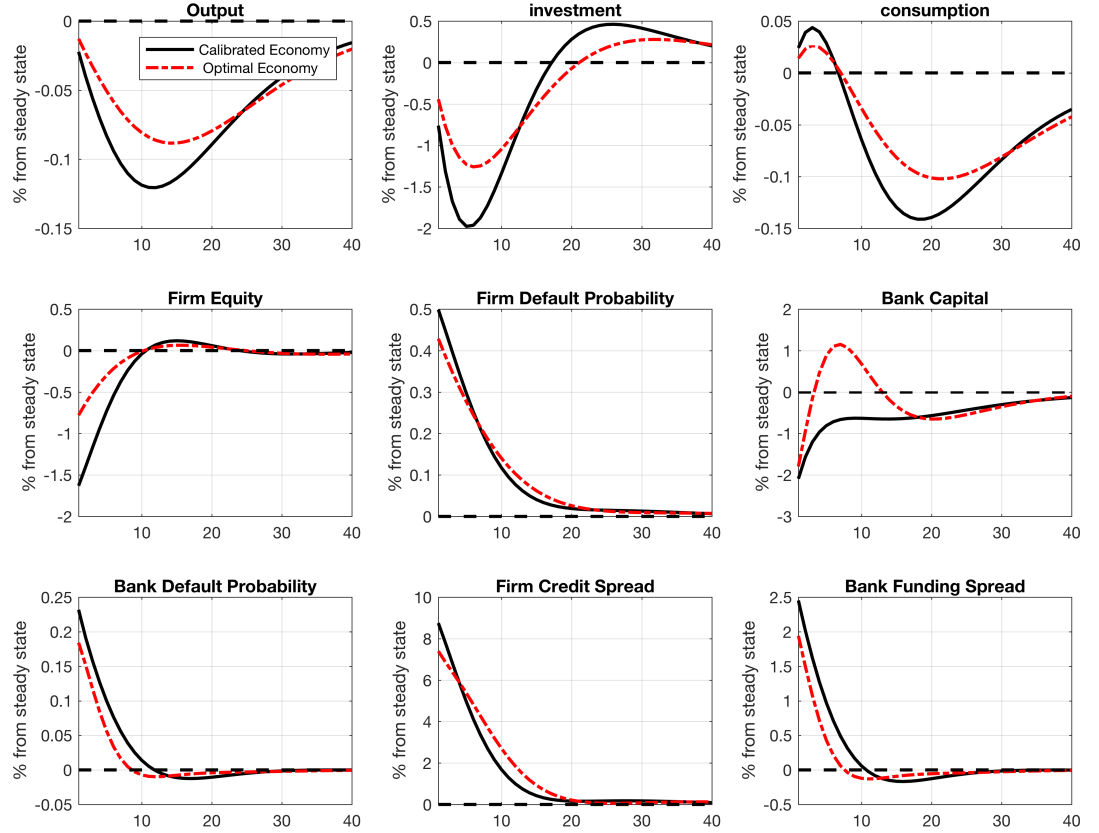
We use the estimated model to simulate responses of four aggregate variables and two key financial variables to shocks. We consider a one standard-deviation increase in the TFP shock, the idiosyncratic risk shock and the aggregate risk shock, respectively. We conduct a counterfactual analysis for each shock by comparing impulse responses of variables in the estimated economy and the optimal economy. The difference between these two economies lie in how firms' and banks' capital structures are determined. In the estimated economy, firm equity and bank equity both follow a law of motion, that is calibrated to match the observed data. In the optimal economy, on the other hand, firms and banks optimally determine their debt/equity levels each period.

Figure (2.4) shows the impulse responses to a one standard-deviation increase in the idiosyncratic risk shock. In the calibrated economy, when idiosyncratic risk rises by one standard deviation, the variance of idiosyncratic productivity increases. Firms' probability of default increases by about 0.5 percentage points. This pushes up firms' credit spread by 6 percentage points. As credit becomes more expensive, firms borrow less and investment subsequently falls by 1 percent on impact and further deteriorates to a trough of about 2 percent. In comparison, the counterfactual economy shows a smaller impact response to the same shock. In particular, firm equity drops but by a lesser extent (about 0.7% compared to 1.6% in the calibrated economy). Firms' default probability and credit spread increase by a relatively smaller amount, although the differences diminish after several quarters. Similarly,

the bank issues equity to buffer the adverse shock. Bank equity rises after the first few quarters, which has a positive impact on bank's default probability and consequently on bank's funding spread. Therefore, the overall impact on output and investment is smaller in the optimal economy.

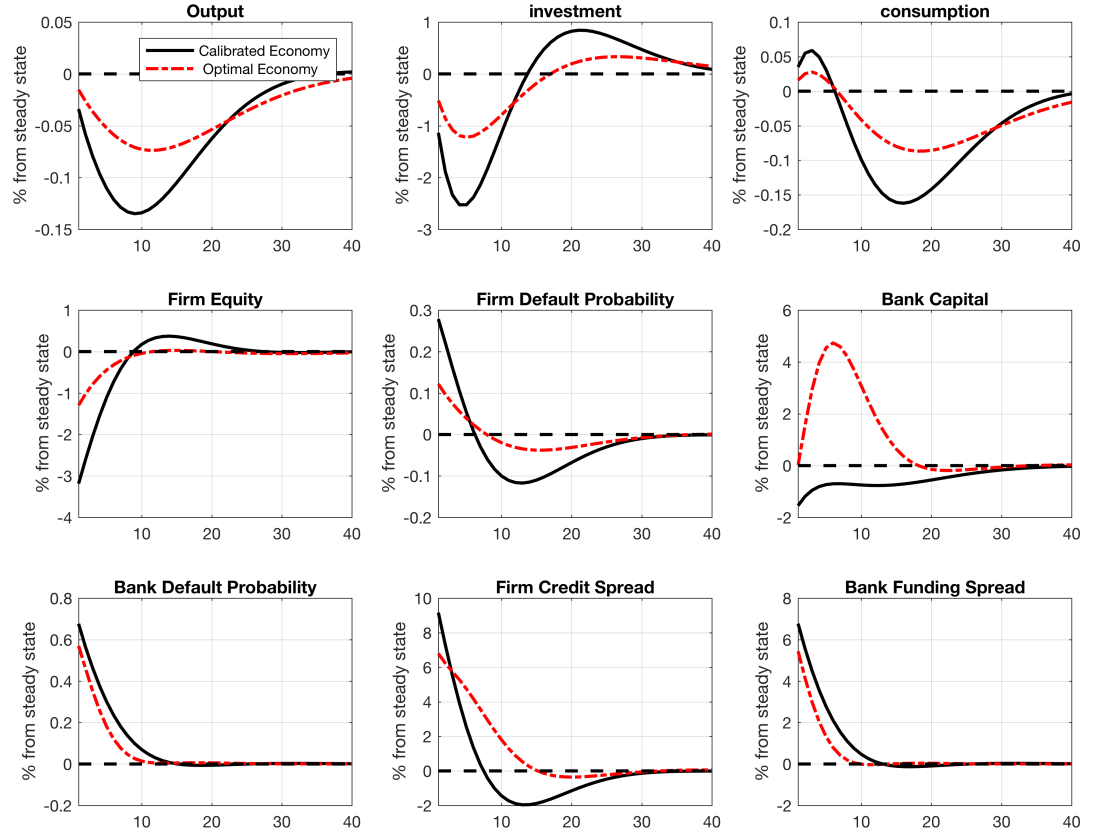
Figure (2.5) shows the impulse responses to a one standard deviation increase in the aggregate risk shock. Following an increase in the aggregate productivity variance, the probability of default for banks increase by 0.65 percentage points. Bank funding spreads rise by about 6.6 percentage points. Higher financial distress is passed to the real economy. Firms' credit spreads abruptly increase by about 9 percentage points, which has a strong adverse impact on the aggregate economy.

We also show the impulse responses to a standard productivity shock in Figure (2.6). The responses of financial variables show a much smaller magnitude compared to the responses to risk shocks. For instance, a one standard-deviation increase in productivity shock raises firms' credit spread by roughly 3 percentage point, and bank funding spread by about 1 percentage point. These two numbers are 9 and 7 percentage points in the aggregate risk shock case, respectively. Additionally, the counterfactual analysis shows the difference between the optimal economy and the calibrated economy is fairly mild in the productivity case, suggesting that capital structures do not matter for the standard productivity shock.



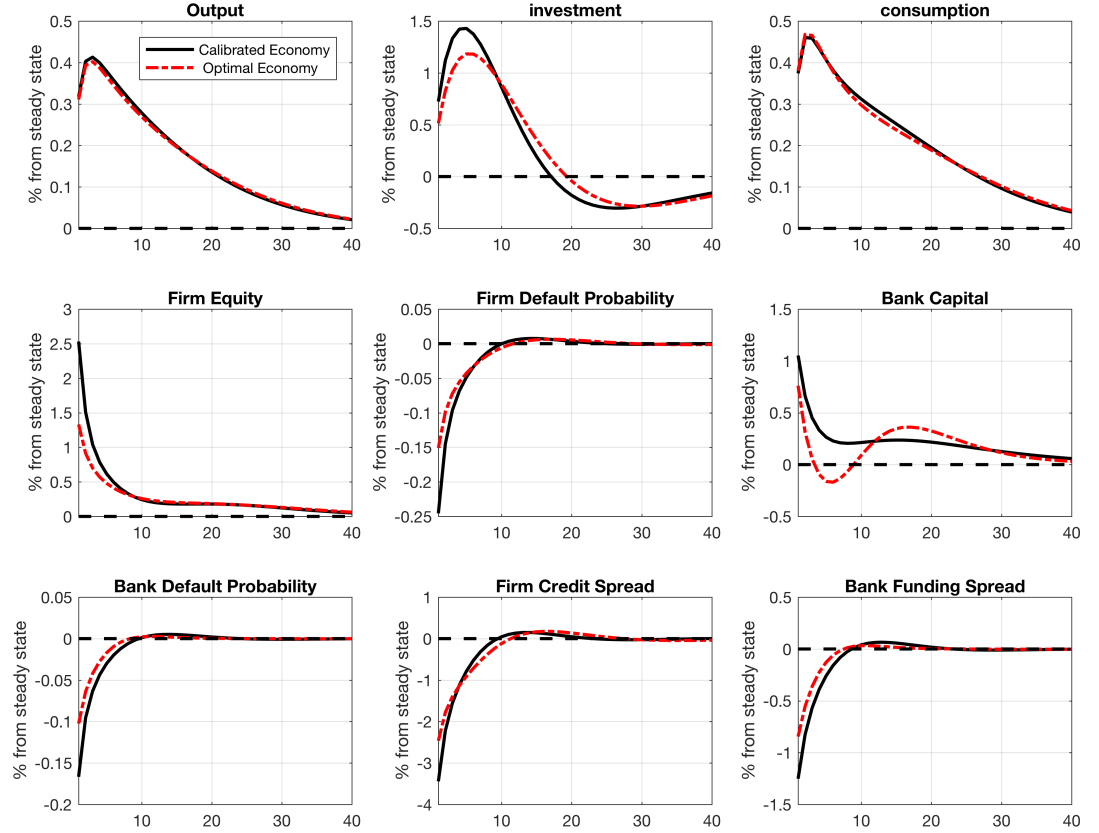
Notes: The figure shows the impulse responses to a one standard-deviation increase in the idiosyncratic risk shock. Values for the persistence and the standard deviation of the shock are based on the posterior mode from the Bayesian estimation. The black solid line represents the estimated economy. The red dash line represents the counterfactual optimal economy where capital structures are optimally chosen. The horizontal axis denotes quarters. The vertical axis denotes the percent (or percentage point) deviation from the steady state value .

Figure 2.4: Impulse Response Functions to Idiosyncratic Risk Shock



Notes: The figure shows the impulse responses to a one standard-deviation increase in the aggregate risk shock. Values for the persistence and the standard deviation of the shock are based on the posterior mode from the Bayesian estimation. The black solid line represents the estimated economy. The red dash line represents the counterfactual optimal economy where capital structures are optimally chosen. The horizontal axis denotes quarters. The vertical axis denotes the percent (or percentage point) deviation from the steady state value .

Figure 2.5: Impulse Response Functions to Aggregate Risk Shock



Notes: The figure shows the impulse responses to a one standard-deviation increase in the productivity shock. Values for the persistence and the standard deviation of the shock are based on the posterior mode from the Bayesian estimation. The black solid line represents the estimated economy. The red dash line represents the counterfactual optimal economy where capital structures are optimally chosen. The horizontal axis denotes quarters. The vertical axis denotes the percent (or percentage point) deviation from the steady state value .

Figure 2.6: Impulse Response Functions to Productivity Shock

2.5.2 Historical Decomposition

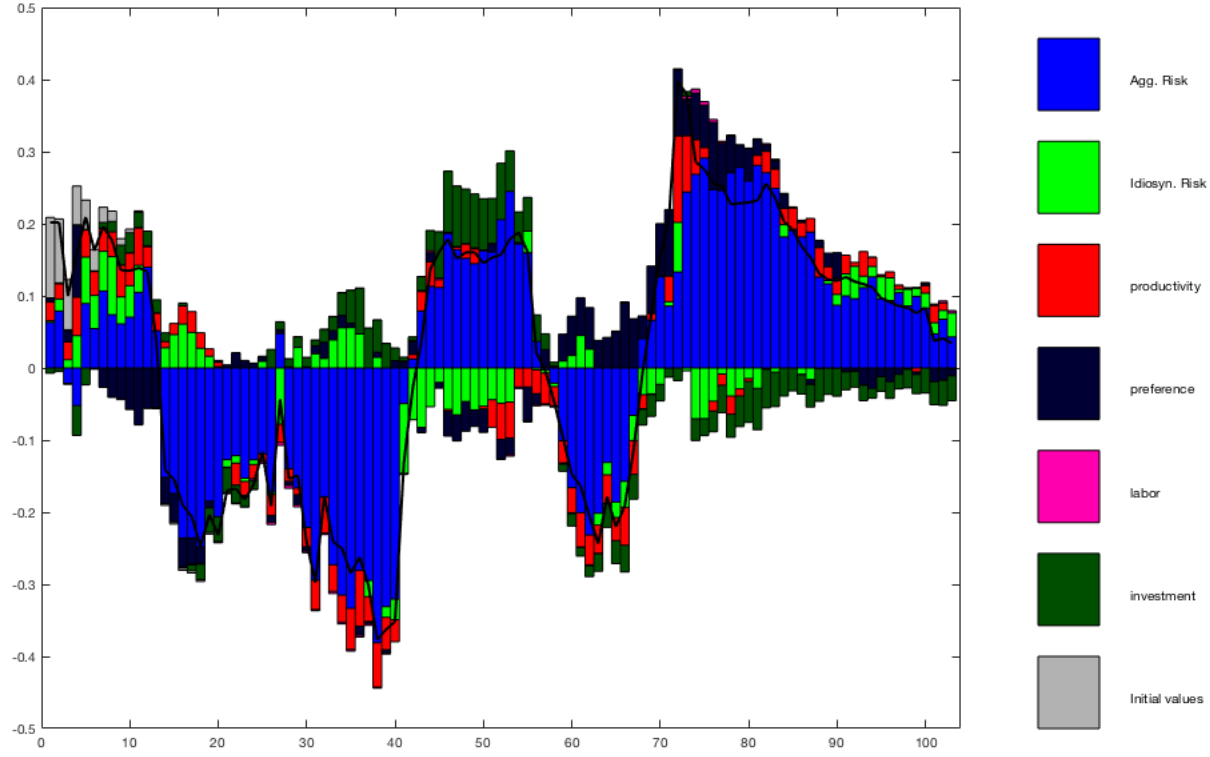
This section describes the historical decomposition of output growth and financial variables into the series of smoothed shocks.

Figure (2.7) shows the time series representation of the evolution of the quarterly change of financial credit spreads in the data. We decompose each quarterly realization (the black line) into the positive (above the x axis) and negative (below the a axis) contributions of the fundamental shocks in the model. The shock series are listed on the right side of the graph. The decomposition suggests that the aggregate risk shock played a significant role in shaping the fluctuations in financial funding spreads, including the spike during the 2008-2009 financial crisis. Figure (2.8) shows the historical decomposition for corporate credit spreads. In this case, both the idiosyncratic risk shock and the productivity shock play relatively larger roles.

We define the bank capital gap as the percentage difference between bank capital in the optimal economy, N_o^B , and bank capital in the estimated economy, N^B :

$$\Delta N^B = \frac{N_o^B - N^B}{N^B} \quad (2.34)$$

Figure (2.9) shows the evolution of the bank capital gap. The gap remains positive during the 2008-2009 financial crisis, and is negative before the crisis. A positive gap means that bank capital in the estimated economy is lower than the optimal level. If banks were to issue equity freely as in the optimal economy, they would increase



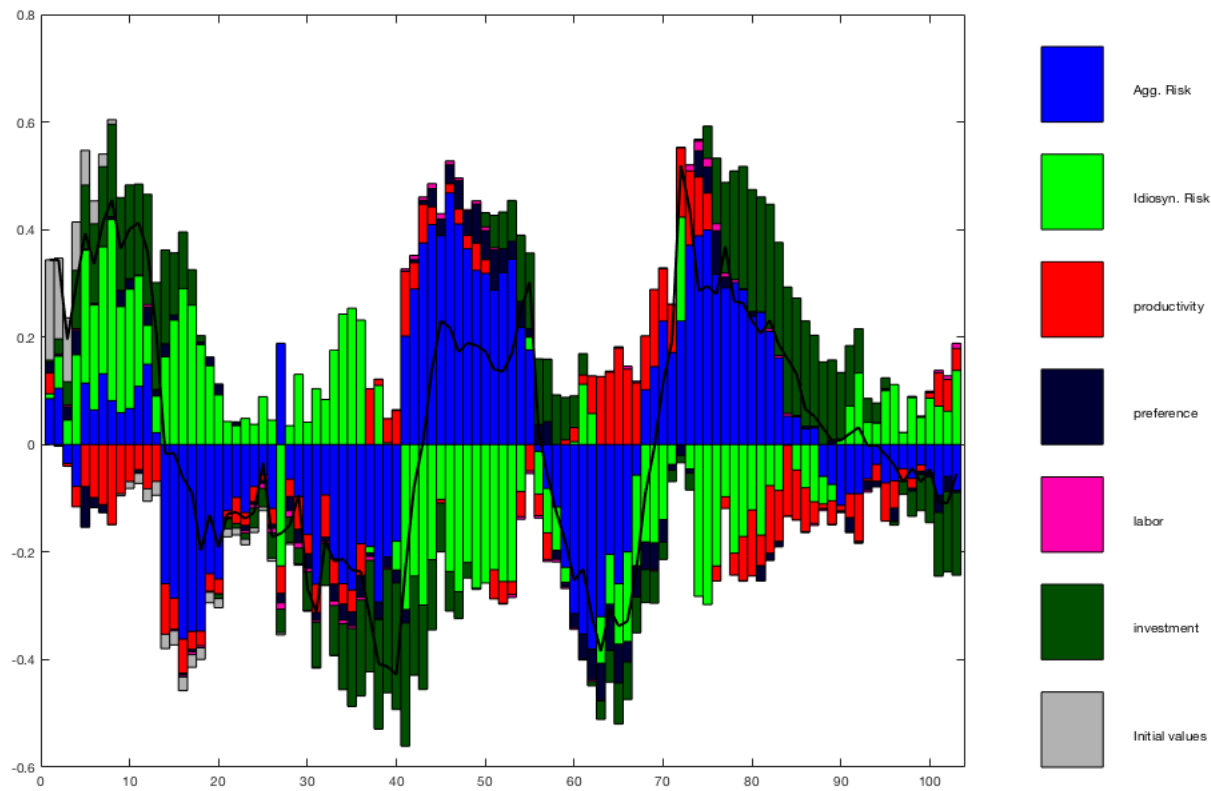
*Notes: Quarterly de-trended financial funding spreads in the data (black line).
Sample period: 1991Q1 - 2016Q4*

Figure 2.7: Historical Decomposition of De-trended Financial Funding Spreads

their capital level. We also show that the aggregate risk shock and the productivity shock are the main drivers of bank capital gaps.

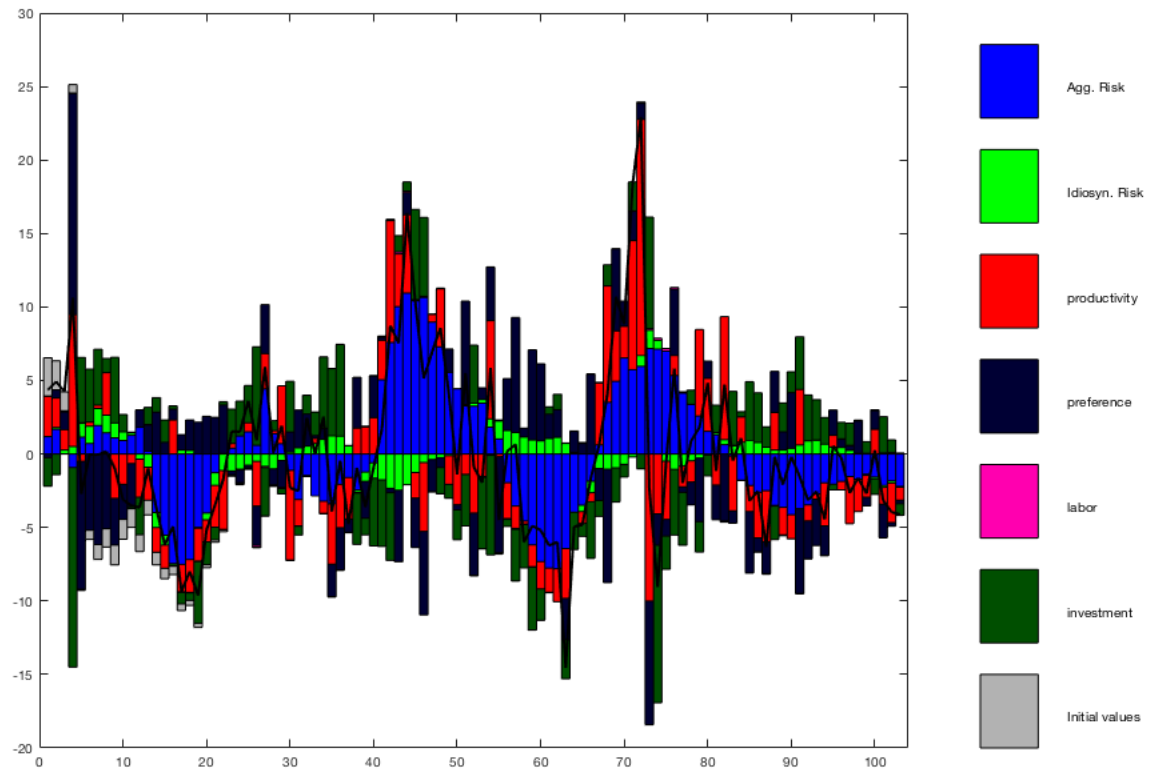
2.6 Conclusion

We develop a quantitative dynamic stochastic general equilibrium model to identify bank capital gaps (deviations of the observed level from the optimum) and to shed light on macro-prudential policies regarding capital requirement. We propose a tractable model to include firms' and banks' joint choice of capital structure, and



*Notes: Quarterly de-trended corporate credit spreads in the data (black line).
Sample period: 1991Q1 - 2016Q4*

Figure 2.8: Historical Decomposition of De-trended Corporate Credit Spreads



Notes: The figure shows

Figure 2.9: Historical Decomposition of Bank Capital Gap

their endogenous default caused by idiosyncratic and aggregate risk. The model is estimated using Bayesian methods with quarterly data on balance sheets and income statements of U.S. financial institutions from 1991 to 2016. We decompose the historical fluctuations in bank capital gaps into contributions from a series of financial shocks, in addition to the standard macroeconomic shocks. We find that the aggregate risk shock plays an important role in explaining the spike in capital gaps during the 2007-09 financial crisis. Capital gaps lead to (i) excessive increases in banks' default risk and cost of funding, (ii) gaps in lending, investment, employment and output.

Appendix A: Appendix for Chapter 1

A.1 Figures of Motivating Facts

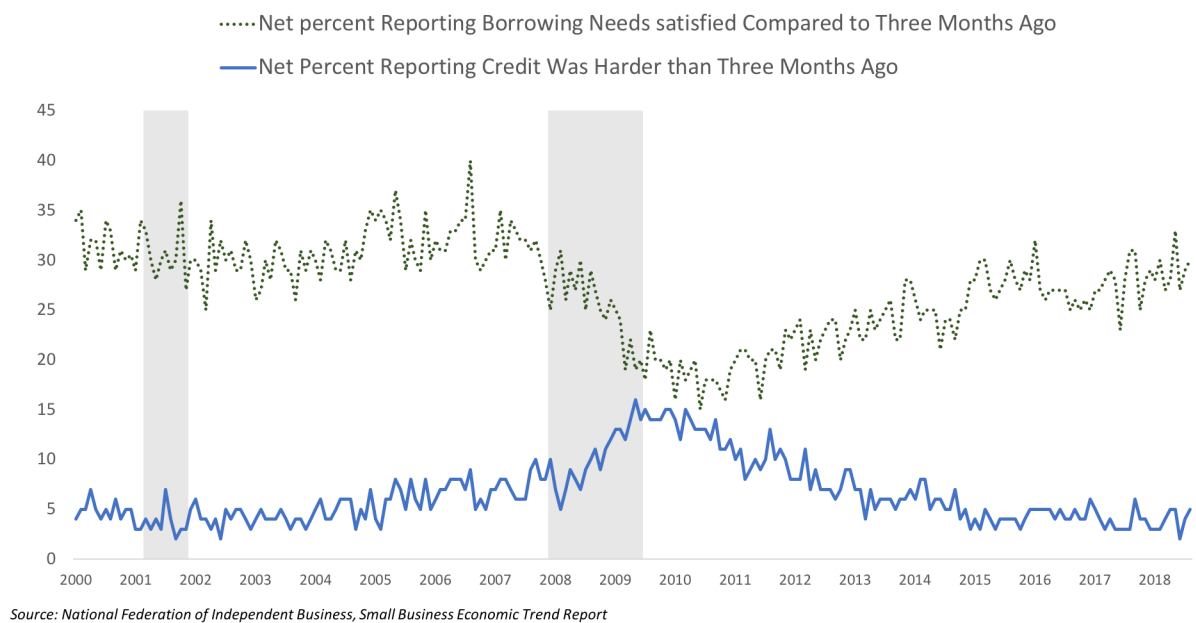
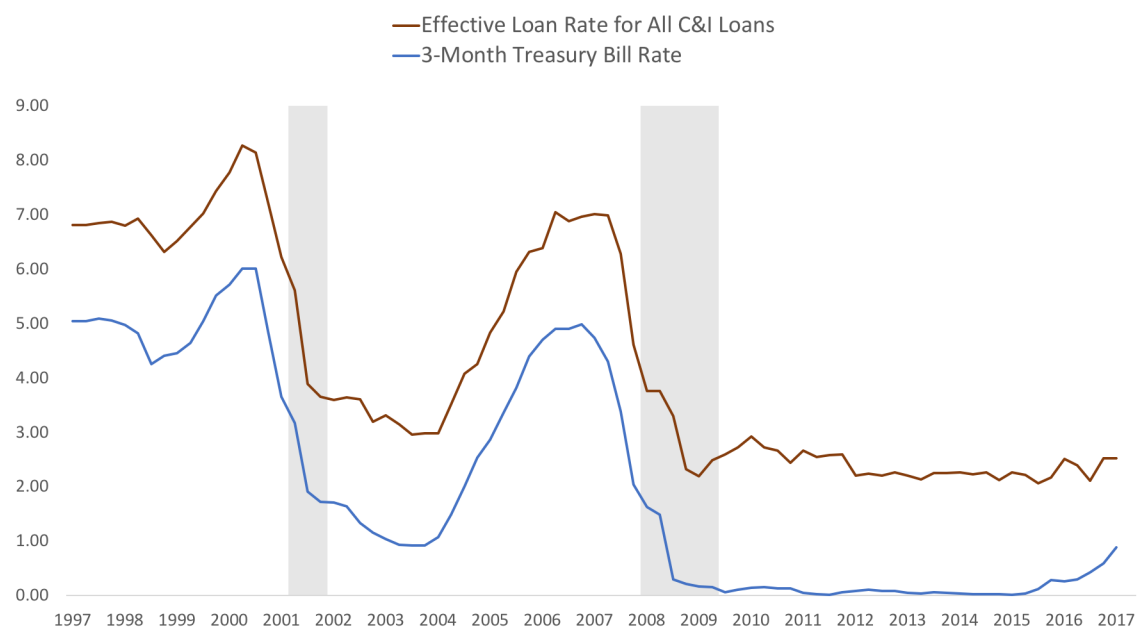


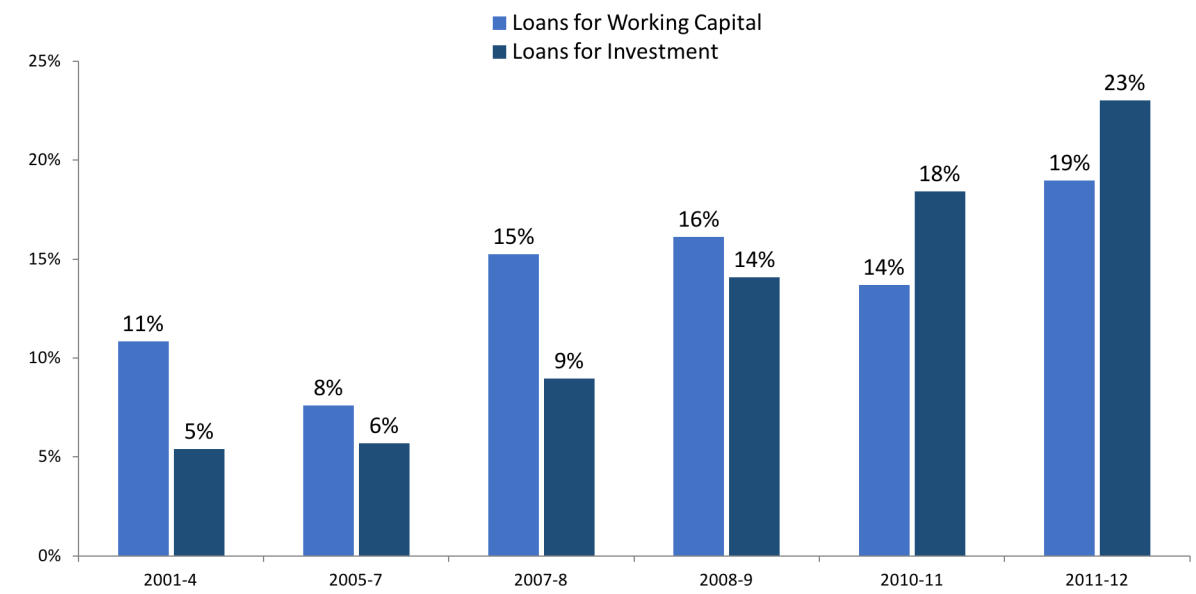
Figure A.1: Measures of credit accessibility for small businesses



Source: Federal Reserve Board, E.2 Survey of Terms of Business Lending

Figure A.2: Interest rates on Commercial and Industrial loans and Treasury Bill

Bank Loan Rejection Rates for U.K. 2001 - 2012



Source: UK Department for Business Innovations and Skills, 2013

Figure A.3: Bank loan rejection rates for U.K. 2001-2012

A.2 Financial Contracts in the Full Model

In this section, I derive optimality conditions characterizing the financial contract. The financial contract is to choose $\{\bar{x}_{t+1}, b_t\}$ to maximize

$$\mathbb{E}_t \left\{ \underbrace{\int_a^{\bar{\sigma}_t} \int_0^\infty \mathcal{S}^E(\bar{x}_{t+1}, z_{t+1}, \sigma_{j,t}) dG(z_{t+1}) dH(\sigma_{j,t} | \sigma_{j,t} < \bar{\sigma}_t)}_{\Psi_{t+1}^A} R_{t+1}^k Q_t K_{t+1}^f \right\}$$

subject to bank's participation constraint

$$\left[\int_{z_{t+1}^*}^\infty \int_a^{\bar{\sigma}_t} \mathcal{S}^B(\bar{x}_{t+1}, z_{t+1}, \sigma_{j,t}) dH(\sigma_{j,t} | \sigma_{j,t} < \bar{\sigma}_t) dG(z_{t+1}) \right] R_{t+1}^k Q_t K_{t+1}^f - (1 - G(z_{t+1}^*)) R_{t+1}^d D_t = R_t N_t^E$$

and mutual fund's participation constraint

$$(1 - G(z_{t+1}^*)) R_{t+1}^d D_t + (1 - \mu_B) \int_0^{z_{t+1}^*} \int_a^{\bar{\sigma}_t} \mathcal{S}^B(\bar{x}_{t+1}, z, \sigma) dH(\sigma | \sigma < \bar{\sigma}_t) dG(z) R_{t+1}^k Q_t K_{t+1}^f \geq R_t D_t$$

The two constraint can be combined as one constraint (with Lagrangian multiplier

λ_{t+1})

$$\underbrace{\left[\int_0^\infty \int_a^{\bar{\sigma}_t} \mathcal{S}^B(\bar{x}_{t+1}, z, \sigma) dH(\sigma | \sigma < \bar{\sigma}_t) dG(z) - \mu_B \int_0^{z_{t+1}^*} \int_a^{\bar{\sigma}_t} \mathcal{S}^B(\bar{x}_{t+1}, z, \sigma) dH(\sigma | \sigma < \bar{\sigma}_t) dG(z) \right]}_{\Psi_{t+1}^B} \\ \times R_{t+1}^k Q_t K_{t+1}^f = R_t (D_t + N_t^B)$$

And bank default threshold also depends on \bar{x}_{t+1} , which is characterized by equation
(with Lagrangian multiplier ν_{t+1})

$$\underbrace{\int_a^{\bar{\sigma}_t} \mathcal{S}^B(\bar{x}_{t+1}, z_{t+1}^*; \sigma_{j,t}) dH(\sigma_{j,t} | \sigma_{j,t} < \bar{\sigma}_t)}_{\Psi_{t+1}^C} R_{t+1}^k Q_t K_{t+1}^f = R_{t+1}^d D_t$$

The first order conditions with respect to b_t , \bar{x}_{t+1} , z_{t+1}^* R_{t+1}^d are given by

$$\Psi_{t+1}^A R_{t+1}^k + \lambda_{t+1} (\Psi_{t+1}^B R_{t+1}^k - R_t) + \nu_{t+1} (\Psi_{t+1}^C R_{t+1}^k - R_{t+1}^d) + \gamma [(1 - G(z^*)) R^d + (1 - \mu_B) \underline{\mathcal{S}}^B(\bar{x}, z^*) R^k - R] =$$

$$\frac{\partial \Psi_{t+1}^A}{\partial \bar{x}_{t+1}} + \lambda_{t+1} \frac{\partial \Psi_{t+1}^B}{\partial \bar{x}_{t+1}} + \nu_{t+1} \frac{\partial \Psi_{t+1}^C}{\partial \bar{x}_{t+1}} + \gamma_{t+1} (1 - \mu_B) \underline{\mathcal{S}}_x^B(\bar{x}_{t+1}, z_{t+1}^*) = 0$$

$$\lambda_{t+1} \mu_B \underline{\mathcal{S}}_z^B(\bar{x}, z^*) = \nu_{t+1} \int_a^{\bar{\sigma}_t} \mathcal{S}_z^B(\bar{x}_{t+1}, z_{t+1}^*; \sigma_{j,t}) dH(\sigma | \sigma < \bar{\sigma}_t) + \gamma [-g(z^*) \frac{R^d D}{R^k Q K^f} + (1 - \mu_B) \underline{\mathcal{S}}_z^B(\bar{x}, z^*)]$$

$$\nu = \gamma(1 - G(z^*))$$

A.3 Optimality Conditions

There are 21 key endogenous variables, $\{R_{t+1}^d, R_{t+1}^F, \bar{x}_t, z_t^*, \bar{x}_t^*, \bar{\sigma}_t, N_t^E, N_t^B, \lambda_t, \nu_t, B_t, D_t, K_t^{sf}, K_t^f, R_t, R_t^k, C_t, L_t, K_t, I_t, Y_t\}$, that are jointly determined by 21 equations listed below.

- Three participation constraints of banks, mutual fund and bank shareholders

$$\text{EShareB}_g(z_t^*, \bar{x}_t, \bar{\sigma}_{t-1})R_t^k Q_{t-1} K_{t-1}^f - (1 - G(z_t^*))R_{t+1}^d D_{t-1} = R_{t-1} N_{t-1}^B \quad (\text{A.1})$$

$$(1 - G(z_t^*))R_{t+1}^d D_{t-1} + (1 - \mu_B)\text{EShareB}_b(z_t^*, \bar{x}_t, \bar{\sigma}_{t-1})R_t^k Q_{t-1} K_{t-1}^f = R_{t-1} D_{t-1} \quad (\text{A.2})$$

$$(1 - G(z_t^*))(R_{t+1}^F B_{t-1} - R_{t+1}^d D_{t-1}) = R_{t-1} N_{t-1}^B \quad (\text{A.3})$$

- Three FOCs from optimal financial contracts

$$\Psi^A(z_{t+1}^*, \bar{x}_{t+1}, \bar{\sigma}_t)R_{t+1}^k Q_t + \lambda_{t+1} (\Psi^B(z_{t+1}^*, \bar{x}_{t+1}, \bar{\sigma}_t)R_{t+1}^k Q_t - R_t) + \nu_{t+1} (\Psi^C(z_{t+1}^*, \bar{x}_{t+1}, \bar{\sigma}_t)R_{t+1}^k Q_t - R_t) = 0 \quad (\text{A.4})$$

$$\Psi_{x,t+1}^A + \lambda_{t+1} \Psi_{x,t+1}^B + \nu_{t+1} \Psi_{x,t+1}^C = 0 \quad (\text{A.5})$$

$$\lambda_{t+1}\mu_B \int_a^{\bar{\sigma}_t} \mathcal{S}^B(\bar{x}_{t+1}, z_{t+1}^*, \sigma) dH(\sigma | \sigma < \bar{\sigma}_t) g(z_{t+1}^*) = \nu_{t+1} \int_a^{\bar{\sigma}_t} \mathcal{S}_z^B(\bar{x}_{t+1}, z_{t+1}^*; \sigma) dH(\sigma | \sigma < \bar{\sigma}_t) \quad (\text{A.6})$$

- Three threshold variables $z_t^*, \bar{x}_t^*, \bar{\sigma}_t$

$$\int_a^{\bar{\sigma}_{t-1}} \mathcal{S}^B(\bar{x}_t, z_t^*, \sigma) dH(\sigma | \sigma < \bar{\sigma}_{t-1}) R_t^k Q_{t-1} K_{t-1}^f = R_{t+1}^d D_{t-1} \quad (\text{A.7})$$

$$\int_0^\infty \mathcal{S}^B(\bar{x}_{t+1}^*, z, \bar{\sigma}_t) dG(z) R_{t+1}^k Q_t K_t^f = R_{t+1}^F B_t \quad (\text{A.8})$$

$$\int_0^\infty \left[(1 - F_{t-1}(\frac{\bar{x}_t^*}{z_t})) \right] dG(z_t) = \mu_E \int_0^\infty \left[f_{t-1}(\frac{\bar{x}_t^*}{z_t}) \frac{\bar{x}_t^*}{z_t} \right] dG(z_t) \quad (\text{A.9})$$

- Law of motions for net worth

$$N_t^E = \gamma^E V_t^E + W_t L^E \quad (\text{A.10})$$

$$N_t^B = \gamma^B V_t^B + W_t L^B \quad (\text{A.11})$$

- Balance sheet identities

$$D_t = B_t - N_t^B \quad (\text{A.12})$$

$$K_t = K_t^f + K_t^{sf} \quad (\text{A.13})$$

$$Q_t K_t^f = B_t + \frac{\bar{\sigma}_{t-1} - a}{b - a} N_t^E \quad (\text{A.14})$$

$$Q_t K_t^{sf} = \frac{b - \bar{\sigma}_{t-1}}{b - a} N_t^E \quad (\text{A.15})$$

- Household utility maximization

$$U_C(C_t, L_t) = \beta \mathbb{E}_t(U_C(C_{t+1}, L_{t+1})R_t) \quad (\text{A.16})$$

$$\frac{U_L(C_t, L_t)}{U_C(C_t, L_t)} = (1 - \alpha)Y_t/L_t \quad (\text{A.17})$$

- Firm profit maximization

$$Y_t = K_t^\alpha (A_t L_t)^{1-\alpha} \quad (\text{A.18})$$

$$K_{t+1} = (1 - \delta)K_t + I_t \quad (\text{A.19})$$

$$R_t^k = \frac{\alpha Y_t / K_t + (1 - \delta)Q_t}{Q_{t-1}} \quad (\text{A.20})$$

- Resource constraints and Market clearing

$$\begin{aligned}
Y_t = & C_t + I_t + \mu_E \int_{z_t^*}^{\infty} \int_a^{\bar{\sigma}_{t-1}} \int_0^{\bar{x}/z} \varepsilon z dF(\varepsilon) dG(z) dH(\sigma | \sigma < \bar{\sigma}_{t-1}) R_t^k K_t^f \\
& + \mu_B \int_0^{z_t^*} \int_a^{\bar{\sigma}_{t-1}} \mathcal{S}^B(\bar{x}_t, z, \sigma) dG(z) dH(\sigma | \sigma < \bar{\sigma}_{t-1}) R_t^k K_t^f
\end{aligned} \tag{A.21}$$

where auxiliary variables are defined as follows

$$\begin{aligned}
\Psi_{t+1}^A &= \int_a^{\bar{\sigma}_t} \int_0^{\infty} \mathcal{S}^E(\bar{x}_{t+1}, z_{t+1}, \sigma_{j,t}) dG(z_{t+1}) dH(\sigma_{j,t} | \sigma_{j,t} < \bar{\sigma}_t) \\
\Psi_{t+1}^B &= \int_0^{\infty} \int_a^{\bar{\sigma}_t} \mathcal{S}^B(\bar{x}_{t+1}, z, \sigma) dH(\sigma | \sigma < \bar{\sigma}_t) dG(z) - \mu_B \int_0^{z_{t+1}^*} \int_a^{\bar{\sigma}_t} \mathcal{S}^B(\bar{x}_{t+1}, z, \sigma) dH(\sigma | \sigma < \bar{\sigma}_t) dG \\
\Psi_{t+1}^C &= \int_a^{\bar{\sigma}_t} \mathcal{S}^B(\bar{x}_{t+1}, z_{t+1}^*; \sigma_{j,t}) dH(\sigma_{j,t} | \sigma_{j,t} < \bar{\sigma}_t) \\
\Psi_{x,t+1}^A &= \int_a^{\bar{\sigma}_t} \int_0^{\infty} \mathcal{S}_x^E(\bar{x}_{t+1}, z_{t+1}, \sigma_{j,t}) dG(z_{t+1}) dH(\sigma_{j,t} | \sigma_{j,t} < \bar{\sigma}_t) \\
\Psi_{x,t+1}^B &= \int_0^{\infty} \int_a^{\bar{\sigma}_t} \mathcal{S}_x^B(\bar{x}_{t+1}, z, \sigma) dH(\sigma | \sigma < \bar{\sigma}_t) dG(z) - \mu_B \int_0^{z_{t+1}^*} \int_a^{\bar{\sigma}_t} \mathcal{S}_x^B(\bar{x}_{t+1}, z, \sigma) dH(\sigma | \sigma < \bar{\sigma}_t) dG \\
\Psi_{x,t+1}^C &= \int_a^{\bar{\sigma}_t} \mathcal{S}_x^B(\bar{x}_{t+1}, z_{t+1}^*; \sigma_{j,t}) dH(\sigma_{j,t} | \sigma_{j,t} < \bar{\sigma}_t)
\end{aligned}$$

$$\mathcal{S}^B(\bar{x}_{t+1}, z_{t+1}, \sigma_{j,t}) = \left((1 - F_{j,t}(\frac{\bar{x}_{t+1}}{z_{t+1}})) \frac{\bar{x}_{t+1}}{z_{t+1}} + (1 - \mu_E) \int_0^{\frac{\bar{x}_{t+1}}{z_{t+1}}} \varepsilon dF_{j,t}(\varepsilon) \right) z_{t+1}$$

$$\tilde{\mathcal{S}}^B(\bar{x}_{t+1}, z_{t+1}, \sigma_{j,t}) = \int_0^{z_{t+1}^*} \int_a^{\bar{\sigma}_t} \mathcal{S}^B(\bar{x}_{t+1}, z, \sigma) dH(\sigma | \sigma < \bar{\sigma}_t) dG(z)$$

$$\mathcal{S}^E(\bar{x}_{t+1}, z_{t+1}, \sigma_{j,t}) = \left(\int_{\frac{\bar{x}_{t+1}}{z_{t+1}}}^{\infty} \varepsilon dF_{j,t}(\varepsilon) - (1 - F_{j,t}(\frac{\bar{x}_{t+1}}{z_{t+1}})) \frac{\bar{x}_{t+1}}{z_{t+1}} \right) z_{t+1}$$

$$\mathcal{S}_x^B(\bar{x}_{t+1}, z_{t+1}, \sigma_{j,t}) = (1 - F_{j,t}(\frac{\bar{x}_{t+1}}{z_{t+1}})) - \mu_E f_{j,t}(\frac{\bar{x}_{t+1}}{z_{t+1}}) \frac{\bar{x}_{t+1}}{z_{t+1}}$$

$$\mathcal{S}_z^B(\bar{x}_{t+1}, z_{t+1}, \sigma_{j,t}) = \mu_E f_{j,t}(\frac{\bar{x}_{t+1}}{z_{t+1}}) \left(\frac{\bar{x}_{t+1}}{z_{t+1}} \right)^2$$

$$\mathcal{S}_x^E(\bar{x}_{t+1}, z_{t+1}, \sigma_{j,t}) = -(1 - F_{j,t}(\frac{\bar{x}_{t+1}}{z_{t+1}}))$$

A.4 Variable Definition

In this section, I list the definition of variables that are used for calibration.

- Loan approval rate:

$$\int_a^{\bar{\sigma}} dH(\sigma) \quad (\text{A.22})$$

- Bank funding credit spread

$$R^F - R \quad (\text{A.23})$$

- Firm borrowing credit spread

$$R^b - R \quad (\text{A.24})$$

- Loss given default (LGD) of bank loans:

$$\begin{aligned} & 1 - \int_a^{\bar{\sigma}} \mathcal{S}^B(\bar{x}, z^*, \sigma) dH(\sigma | \sigma < \bar{\sigma}) \\ &= 1 - \int_a^{\bar{\sigma}} \left((1 - F(\frac{\bar{x}}{z^*}; \sigma)) \frac{\bar{x}}{z^*} + (1 - \mu_E) \int_0^{\frac{\bar{x}}{z^*}} \varepsilon dF(\varepsilon; \sigma) \right) z^* dH(\sigma | \sigma < \bar{\sigma}) \end{aligned} \quad (\text{A.25})$$

- Firms' probability of default (PD)

$$\int_a^{\bar{\sigma}} F(\frac{\bar{x}}{z^*}; \sigma_j) dH(\sigma_j | \sigma_j < \bar{\sigma}) \quad (\text{A.26})$$

- Banks' probability of default (PD)

$$G(z^*) \tag{A.27}$$

- Firm asset-to-equity ratio

$$\frac{K^f}{K^f - B} \tag{A.28}$$

- Bank debt-to-equity ratio

$$\frac{D}{N^B} \tag{A.29}$$

Appendix A: Appendix for Chapter 2

A.1 Equilibrium Conditions for Optimal Economy

The first order condition K_{t+1}

$$\begin{aligned} \mathbb{E}_t \left\{ M_{t+1} \mathcal{S}^E(\bar{x}) R_{t+1}^k \right\} (1 - \tau) - 1 + \lambda \mathbb{E}_t \left\{ M_{t+1} \left[\bar{\mathcal{S}}^B(\bar{x}_{t+1}, z_{t+1}^*) R_{t+1}^k \right] \right\} (1 - \tau) \\ + \gamma \mathbb{E}_t \left\{ M_{t+1} [(1 - \mu_B) \underline{\mathcal{S}}^B(\bar{x}, z^*) R^k] \right\} + \nu \mathbb{E} M_{t+1} \mathcal{S}^B(\bar{x}, z^*) R^k = 0 \end{aligned} \quad (\text{A.1})$$

We can write it differently

$$\mathbb{E}_t \left[M_{t+1} \Lambda_{t+1} R_{t+1}^k \right] = 1 \quad (\text{A.2})$$

where

$$\Lambda_{t+1} = (\mathcal{S}^E(\bar{x}) + \lambda \bar{\mathcal{S}}^B(\bar{x}_{t+1}, z_{t+1}^*)) (1 - \tau) + \gamma (1 - \mu_B) \underline{\mathcal{S}}^B(\bar{x}, z^*) + \nu \mathcal{S}^B(\bar{x}, z^*) \quad (\text{A.3})$$

FOC \bar{x}_{t+1}

$$\begin{aligned} \mathbb{E}_t M_{t+1} [\mathcal{S}_x^E(\bar{x})(1-\tau) + \lambda \bar{\mathcal{S}}_x^B(\bar{x}_{t+1}, z_{t+1}^*)(1-\tau) \\ + \gamma(1-\mu_B) \underline{\mathcal{S}}_x^B(\bar{x}, z^*) + \nu \mathcal{S}_x^B(\bar{x}, z^*)] = 0 \end{aligned} \quad (\text{A.4})$$

FOC z_{t+1}^*

$$\begin{aligned} \mathbb{E}_t M_{t+1} [[\lambda \bar{\mathcal{S}}_z^B(\bar{x}, z^*)(1-\tau) + \gamma(1-\mu_B) \underline{\mathcal{S}}_z^B(\bar{x}, z^*) + \nu \mathcal{S}_z^B(\bar{x}, z^*)] R^k Q K \\ + [\lambda g(z^*)(1-\tau) - \gamma g(z^*)] R^d D] = 0 \end{aligned} \quad (\text{A.5})$$

FOC R_{t+1}^d

$$E_t M_{t+1} [(-\lambda(1-\tau) + \gamma)(1 - G(z^*)) - \nu] = 0 \quad (\text{A.6})$$

FOC B_t

$$\lambda = 1 \quad (\text{A.7})$$

FOC D_t

$$\lambda(\mathbb{E} M_{t+1} [-(1 - G(z^*)) R^d (1-\tau)] + 1) + \gamma(\mathbb{E} M_{t+1} (1 - G(z^*)) R^d - 1) - \nu \mathbb{E} M_{t+1} R^d = 0 \quad (\text{A.8})$$

A.2 Equilibrium Conditions in the Calibrated Economy

The first order conditions with respect to K_{t+1} , \bar{x}_{t+1} , z_{t+1}^* and R_{t+1}^d are given by¹

¹Here time subscripts are omitted for compact notation.

FOC K_{t+1}

$$\begin{aligned} & \mathcal{S}^E(\bar{x})R^k + \gamma \left[(1 - G(z^*))R^d + (1 - \mu_B)\underline{\mathcal{S}}^B(\bar{x}, z^*)R^k - R \right] \\ & + \lambda \left[(\bar{\mathcal{S}}^B(\bar{x}, z^*) + (1 - \mu_B)\underline{\mathcal{S}}^B(\bar{x}, z^*))R^k - R \right] + \nu[\mathcal{S}^B(\bar{x}, z^*)R^k - R^d] = 0 \end{aligned} \quad (\text{A.9})$$

FOC \bar{x}_{t+1}

$$\mathcal{S}_x^E(\bar{x}) + \lambda \left[\bar{\mathcal{S}}_x^B(\bar{x}, z^*) + (1 - \mu_B)\underline{\mathcal{S}}_x^B(\bar{x}, z^*) \right] + \gamma(1 - \mu_B)\underline{\mathcal{S}}_x^B(\bar{x}, z^*) + \nu\mathcal{S}_x^B(\bar{x}, z^*) = 0 \quad (\text{A.10})$$

FOC z_{t+1}^*

$$\begin{aligned} & \gamma_{t+1} \left[-g(z_{t+1}^*)R_{t+1}^d D_t + (1 - \mu_B)\mathcal{S}^B(\bar{x}, z^*)g(z^*)R^k QK \right] \\ & - \lambda_{t+1}\mu_B\mathcal{S}^B(\bar{x}, z^*)g(z^*)R^k QK + \nu\mathcal{S}_z^B(\bar{x}, z^*)R^k QK = 0 \end{aligned} \quad (\text{A.11})$$

FOC R_{t+1}^d

$$\nu = \gamma(1 - G(z^*)) \quad (\text{A.12})$$

A.2.1 Equilibrium Conditions

There are 15 key endogenous variables:

$$\bar{x}_t, z_t^*, \lambda_t, \nu_t, R_{t+1}^d, N_t^E, N_t^B, C_t, L_t, K_t, I_t, Y_t, R_t, R_t^k, D_t$$

The first eight equations jointly determine the set of eight financial contractual variables $\{\bar{x}_t, z_t^*, \lambda_t, \nu_t, R_{t+1}^d, D_t\}$. The standard RBC model is given by equation

(A.21) - (A.26), plus the standard investment Euler equation $E_t(M_{t+1}R_{t+1}^k) = 1$, which jointly determine the path of 7 endogenous variables $\{C_t, L_t, K_t, I_t, Y_t, R_t^k, R_t\}$.

- Participation constraints of banks and the mutual fund

$$\left[\int_{z_{t+1}^*}^{\infty} \mathcal{S}^B(\bar{x}_{t+1}, z_{t+1}) dG(z_{t+1}) \right] R_{t+1}^k Q_t K_{t+1} - (1 - G(z_{t+1}^*)) R_{t+1}^d D_t = R_t N_t^B \quad (\text{A.13})$$

$$(1 - G(z_{t+1}^*)) R_{t+1}^d D_t + (1 - \mu_B) \int_0^{z_{t+1}^*} \mathcal{S}^B(\bar{x}_{t+1}, z) dG(z) R_{t+1}^k Q_t K_{t+1} \geq R_t D_t \quad (\text{A.14})$$

- Bank default threshold z_{t+1}^*

$$\mathcal{S}^B(\bar{x}_{t+1}, z_{t+1}^*) R_{t+1}^k Q_t K_{t+1} = R_{t+1}^d D_t \quad (\text{A.15})$$

- Optimality conditions of financial contracts

$$\Psi_{t+1}^A R_{t+1}^k + \lambda_{t+1} (\Psi_{t+1}^B R_{t+1}^k - R_t) + \nu_{t+1} (\Psi_{t+1}^C R_{t+1}^k - R_{t+1}^d) = 0 \quad (\text{A.16})$$

$$\frac{\partial \Psi_{t+1}^A}{\partial \bar{x}_{t+1}} + \lambda_{t+1} \frac{\partial \Psi_{t+1}^B}{\partial \bar{x}_{t+1}} + \nu_{t+1} \frac{\partial \Psi_{t+1}^C}{\partial \bar{x}_{t+1}} = 0 \quad (\text{A.17})$$

$$\lambda_{t+1} \mu_B \mathcal{S}^B(\bar{x}_{t+1}, z_{t+1}^*) g(z_{t+1}^*) = \nu_{t+1} \mathcal{S}_z^B(\bar{x}_{t+1}, z_{t+1}^*) \quad (\text{A.18})$$

- Law of motion equities

$$N_t^E = \gamma^E V_t^E + W_t L^E \quad (\text{A.19})$$

$$N_t^B = \gamma^B V_t^B + W_t L^B + T_t \quad (\text{A.20})$$

- Household inter-temporal and intra-temporal optimality conditions

$$(1 - \alpha)Y_t/L_t = \psi_L L_t^\varphi C_t \quad (\text{A.21})$$

$$\beta \mathbb{E}_t \left(\frac{U_C(t+1)}{U_C(t)} R_t \right) = 1 \quad (\text{A.22})$$

- Firms' optimality conditions

$$Y_t = K_{t-1}^\alpha (A_t L_t)^{1-\alpha} \quad (\text{A.23})$$

$$I_t = K_t - (1 - \delta)K_{t-1} \quad (\text{A.24})$$

$$R_t^k = \alpha Y_t / K_{t-1} + 1 - \delta \quad (\text{A.25})$$

- Resource constraint

$$C_t + I_t = Y_t \tag{A.26}$$

- Balance sheet accounting identity

$$D_t = K_t - N_t^E - N_t^B \tag{A.27}$$

Bibliography

- Adrian, T., Colla, P., and Song Shin, H. (2013). Which Financial Frictions? Parsing the Evidence from the Financial Crisis of 2007 to 2009. *NBER Macroeconomics Annual*, 27(1):159–214.
- Ajello, A. and Tanaka, H. (2017). Term premium, credit risk premium, and monetary policy. Technical report, Mimeo, Board of Governors of the Federal Reserve System.
- Bentolila, S., Jansen, M., and Jiménez, G. (2018). When Credit Dries Up: Job Losses in the Great Recession. *Journal of the European Economic Association*, 16(3):650–695.
- Bernanke, B. S., Gertler, M., and Gilchrist, S. (1999). The financial accelerator in a quantitative business cycle framework. *Handbook of macroeconomics*, 1:1341–1393.
- Bester, H. (1985). Screening vs. rationing in credit markets with imperfect information. *The American Economic Review*, 75(4):850–855.
- Bigio, S. (2015). Endogenous Liquidity and the Business Cycle. *American Economic Review*, 105(6):1883–1927.
- Brunnermeier, M. K. and Sannikov, Y. (2014). A Macroeconomic Model with a Financial Sector. *American Economic Review*, 104(2):379–421.
- Carlstrom, C. T. and Fuerst, T. S. (1997). Agency costs, net worth, and business fluctuations: A computable general equilibrium analysis. *The American Economic Review*, pages 893–910.
- Chodorow-Reich, G. (2014). The Employment Effects of Credit Market Disruptions: Firm-level Evidence from the 2008–9 Financial Crisis. *The Quarterly Journal of Economics*, 129(1):1–59.
- Christiano, L. J., Eichenbaum, M., and Evans, C. L. (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of political Economy*, 113(1):1–45.

- Christiano, L. J., Motto, R., and Rostagno, M. (2014). Risk Shocks. *American Economic Review*, 104(1):27–65.
- Clerc, L., Derviz, A., Mendicino, C., Moyen, S., Nikolov, K., Stracca, L., Suarez, J., and Vardoulakish, A. P. (2015). Capital Regulation in a Macroeconomic Model with Three Layers of Default. *International Journal of Central Banking*.
- Comin, D. and Gertler, M. (2006). Medium-term business cycles. *American Economic Review*, 96(3):523–551.
- Daruich, D. (2018). The Macroeconomic Consequences of Early Childhood Development Policies. URL: <https://drive.google.com/file/d/0B5YmjVfr68oLbUUMWI4clk4NXc/view>.
- De Fiore, F. and Uhlig, H. (2011). Bank finance versus bond finance. *Journal of Money, Credit and Banking*, 43(7):1399–1421.
- Del Negro, M., Eggertsson, G. B., Ferrero, A., and Kiyotaki, N. (2011). The great escape? A quantitative evaluation of the Fed’s liquidity facilities. *A Quantitative Evaluation of the Fed’s Liquidity Facilities (October 1, 2011)*. FRB of New York Staff Report, (520).
- Dell’ariccia, G. and Marquez, R. (2006). Lending Booms and Lending Standards. *The Journal of Finance*, 61(5):2511–2546.
- Figueroa, N. and Leukhina, O. (2015). Lending terms and aggregate productivity. *Journal of Economic Dynamics and Control*, 59:1–21.
- Gertler, M. and Karadi, P. (2011). A model of unconventional monetary policy. *Journal of Monetary Economics*, 58(1):17–34.
- Gertler, M. and Kiyotaki, N. (2010). Financial intermediation and credit policy in business cycle analysis. *Handbook of monetary economics*, 3(3):547–599.
- Gertler, M. and Kiyotaki, N. (2015). Banking, Liquidity, and Bank Runs in an Infinite Horizon Economy. *American Economic Review*, 105(7):2011–2043.
- Gete, P. (2018). Lending Standards and Macroeconomic Dynamics. Working Paper.
- Gourio, F. (2013). Credit risk and disaster risk. *American Economic Journal: Macroeconomics*, 5(3):1–34.
- He, Z. and Krishnamurthy, A. (2013). Intermediary Asset Pricing. *American Economic Review*, 103(2):732–770.
- Hu, Y. (2017). *Recovery Dynamics: An Explanation From Bank Lending and Entrepreneur Entry*. PhD thesis, The University of Chicago.
- Iacoviello, M. (2015). Financial business cycles. *Review of Economic Dynamics*, 18(1):140–163.

- Ivashina, V. and Scharfstein, D. (2010). Bank lending during the financial crisis of 2008. *Journal of Financial economics*, 97(3):319–338.
- Jaffee, D. M. and Russell, T. (1976). Imperfect information, uncertainty, and credit rationing. *The Quarterly Journal of Economics*, 90(4):651–666.
- Jermann, U. and Quadrini, V. (2012). Macroeconomic Effects of Financial Shocks. *American Economic Review*, 102(1):238–271.
- Jiménez, G., Moral-Benito, E., and Vegas, R. (2018). Bank Lending Standards Over the Cycle: The Role of Firms’ Productivity and Credit Risk. SSRN Scholarly Paper ID 3163419, Social Science Research Network, Rochester, NY.
- Kiyotaki, N. and Moore, J. (1997). Credit Cycles. *Journal of Political Economy*, 105(2):211–248.
- Laufer, S. and Paciorek, A. (2016). The effects of mortgage credit availability: Evidence from minimum credit score lending rules.
- Montoriol-Garriga, J. and Wang, J. C. (2011). The great recession and bank lending to small businesses. Technical report, Working paper series//Federal Reserve Bank of Boston.
- Nuño, G. and Thomas, C. (2017). Bank leverage cycles. *American Economic Journal: Macroeconomics*, 9(2):32–72.
- Ravn, S. H. (2016). Endogenous credit standards and aggregate fluctuations. *Journal of Economic Dynamics and Control*, 69:89–111.
- Rodano, G., Serrano-Velarde, N., and Tarantino, E. (2018). Lending Standards over the Credit Cycle. *The Review of Financial Studies*, 31(8):2943–2982.
- Ruckes, M. (2004). Bank competition and credit standards. *Review of Financial Studies*, 17(4):1073–1102.
- Shibut, L. and Singer, R. (2015). Loss Given Default for Commercial Loans at Failed Banks.
- Smets, F. and Wouters, R. (2007). Shocks and frictions in US business cycles: A Bayesian DSGE approach. *American economic review*, 97(3):586–606.
- Stiglitz, J. E. and Weiss, A. (1981). Credit rationing in markets with imperfect information. *The American economic review*, pages 393–410.
- Williamson, S. D. (1987). Costly monitoring, loan contracts, and equilibrium credit rationing. *The Quarterly Journal of Economics*, 102(1):135–145.